

Algebra II, Block G
September 26, 2011
Linear Programming

1. Hiring Teachers

A School Board is investigating various ways of composing the faculty for a proposed new elementary school. They can hire teachers and aides. The amount of money the school district will have to spend on salaries each year depends on how many teachers and on how many aides are hired.

Let t = number of teachers hired.
Let a = number of aides hired.
Let d = number of thousands of dollars spent
annually on faculty salaries.

(a) The School Board finds that there are a few requirements concerning the number of teachers and aides that it may hire; these are listed below. For each requirement, write an equality or inequality using the variables t , and a to describe the requirement mathematically.

i. The building can accommodate no more than 50 faculty members, total.

ii. A minimum of 20 faculty members is needed to staff the school.

iii. The school cannot be run entirely by aides, so there must be at least 12 teachers.

- iv. For a proper teacher to aide ratio, the number of teachers must be at least half the number of aides.

- v. It is impossible (obviously!) to hire a *negative* number of teachers or aides.

- (b) That was a lot of requirements to keep track of, so let's try organizing them visually. Draw all of the inequalities you just wrote on a single graph. Be sure to label your axes and each feature on your graph appropriately. Shade the region on your graph that corresponds to values of a and t that satisfy all of the Board's requirements; this is called the *feasible* region. Do this on a separate piece of graph paper.

- (c) The Board finds that the average teacher's annual salary is \$30,000, and the average aide's annual salary is \$20,000. Write an equation that gives d as a function of t and a .

- (d) The Board decides that the most they can spend on staffing this school is \$600,000.
 - i. Write an (in)equality that represents this requirement.

 - ii. Add this new requirement to your graph, and shade the new feasible region.

- iii. Suggest a number of teachers and aides that meet all of the boards requirements. What is the cost associated with this staffing level?
- (e) The board would like to know if they can staff the school for \$400,000 or less. Write an inequality for this requirement, and add it to your graph. Can they staff the school for \$400,000 or less?
- (f) Can the board staff the school for \$200,000 or less? Explain your reasoning.
- (g) What is the least amount of money the board can spend on staffing this school while still meeting all of their requirements? How many aides and how many teachers would they hire at this cost?