Section 7.4: More Factoring and Graphing

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Warm-Up

Please work on the following problems as a warm-up:

1. Multiply:
   $(x^2 - 2x + 5)(x - 3)$
2. Factor completely:
   $6x^2 + 19x - 20$
3. Factor completely using the difference of two squares:
   $k^7 - k^3$
4. Multiply:
   $(x + y)(x^2 - xy + y^2)$

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Factoring a Sum of Two Cubes

In the warm-up problems, we showed that

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2).$$

We can think of this as a rule for factoring, and use it when we need to factor the sum of two cubes. For example, factor

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$$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$$

Factoring a Difference of Two Cubes

We now know that

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2).$$

Can you find an equivalent expression for $x^3 - y^3$?

Hint:

$$\begin{aligned} x^3 - y^3 &= x^3 + (-y)^3 \\ &= ? \end{aligned}$$

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Factoring a Sum or Difference of Two Cubes

Write these equations in your notes: you'll need them!

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Factoring a Sum or Difference of Two Cubes: Examples

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Factor the polynomial completely:

1. $c^3 - 729$
2. $p^3 + w^{12}$
3. $3c^4 - 81c$

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3. $3c^4 - 81c = 3c(c - 3)(c^2 + 3c + 9)$

Roots and Factors of Quadratics

Determine the roots of the quadratic functions below, then factor them.

Remember, a is a root of the function $f(x)$ if and only if ________.

1. $p(x) = x^2 - 7x + 12$
2. $g(x) = x^2 + 3x - 10$
3. $m(x) = 6x^2 + 19x - 20$

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Determine the roots of the quadratic functions below, then factor them.

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1. $p(x) = x^2 - 7x + 12 = (x-3)(x-4)$. $p(x)$ has roots 3 and 4.
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2. $g(x) = x^2 + 3x - 10 = (x+5)(x-2)$. $g(x)$ has roots -5 and 2.
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2. $g(x) = x^2 + 3x - 10 = (x+5)(x-2)$. $g(x)$ has roots -5 and 2.
3. $m(x) = 6x^2 + 19x - 20 = (6x-5)(x+4)$. $m(x)$ has roots $\frac{5}{6}$ and -4.

Can you state a relationship between the factors of a quadratic and the roots?

Roots and Factors of Polynomials

If a is a root of the polynomial $p(x)$, then $(x - a)$ is a factor of $p(x)$.

Example: Factor the polynomial $f(x) = x^2 - 21x + 68$.

First, find the roots of $f(x)$:

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First, find the roots of $f(x)$: 17 and 4.

Thus, $f(x)$ must have two factors: $(x - 17)$ and $(x - 4)$.

Let's guess that $f(x) = (x - 17)(x - 4)$. Confirm this factoring is correct by expanding the terms.

Roots and Factors of Polynomials

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First, find the roots of $f(x)$: 1 and $\frac{13}{3}$.

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$(x-1)(x-\frac{13}{3}) = x^2 - \frac{16}{3}x + \frac{13}{3} \neq 3x^2 - 16x + 13.$

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$(x - 1)(x - \frac{13}{3}) = x^2 - \frac{16}{3}x + \frac{13}{3} \neq 3x^2 - 16x + 13$. If we multiply be 3, the coefficient of the leading term, everything does work:

$$f(x) = 3(x - 1)\left(x - \frac{13}{3}\right) = (x - 1)(3x - 13).$$

Roots and Factors of Polynomials

To factor a polynomial $p(x)$ with leading coefficient c you can:

1. Find the roots of $p(x)$. Let's call them $a_0, a_1, \ldots, a_n$.
2. Then $p(x) = c(x - a_0)(x - a_1)\ldots(x - a_n)$.

Examples: Factor completely:

1. $f(x) = 20x^2 + 89x + 18$
2. $f(x) = x^2 + 1$
3. $f(x) = x^2 - 2$