

Rational Functions and Black Holes

Name: _____

Physicists call the speed of light in a vacuum c . It is approximately

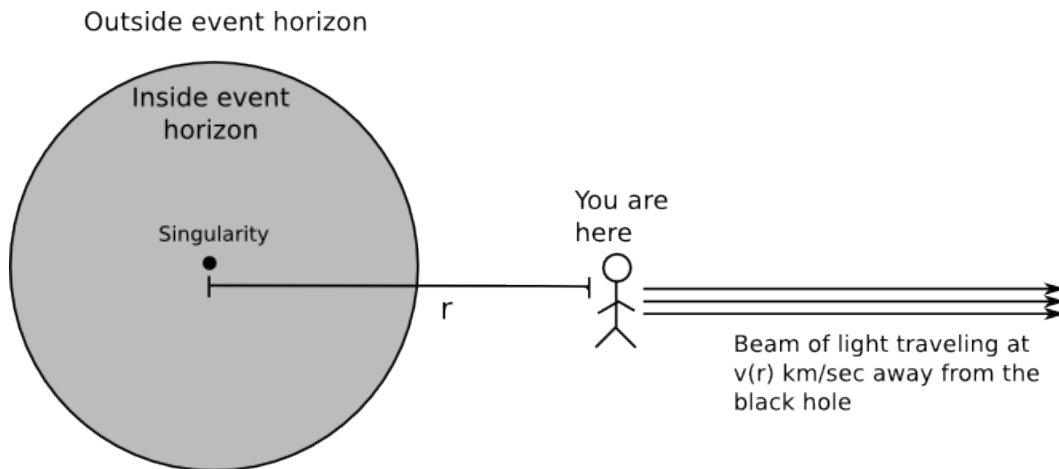
$$c \approx 300,000 \text{ km/sec}$$

This is fast enough to go around the earth more than seven times in one second!¹ However, light moves slower when it goes through air, water, or other material. Light can also be slowed down by gravity of very massive objects like collapsed stars. For the example today we'll consider Cygnus X-1, a black hole formed by a star with 9 times the mass of our Sun.

If we are r km from the center of the black hole and we shine a light away from the black hole, according to general relativity the speed of the light will be approximately

$$v(r) = c \left(1 - \frac{26}{r} \right)$$

Here is a picture of the situation:



1. What is the speed of our beam of light if we are 13 km from the center of the black hole? What does this mean?

¹This is one of the reasons fiber optics are used for backbone of the internet: light moves so much faster than electricity.

2. The event horizon is the region around the center of the black hole where gravity is so strong that even light can't escape (it's the reason we call them black holes). At what radius is the event horizon?
3. What are the horizontal asymptotes of $v(r)$? What is the **physical interpretation** of the horizontal asymptote?
4. What are the vertical asymptotes of $v(r)$? Give a **physical interpretation** of the vertical asymptote.
5. Use your calculator to sketch a graph of $v(r)$. You should think about what a reasonable domain would be. Your graph should have numbers on both axes and indicate the units. Finding a suitable window may not be easy, you should think about the values of r (the domain) and the values of v (the range).
6. Does the velocity ever equal $-2c$? If so, where?
7. What do you think happens to the velocity as r approaches 0?