

Logarithms

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1 Phonebook Search

1.1 Intro

1. The students have just been introduced to logarithms
2. Instead of “talking” about what logarithms are for or what they model, we will have them discover it.
3. This will be done through a phonebook search.

When the students search a phonebook, they will be unconsciously doing logarithmic search (or a similar algorithm called phonebook search). The key idea is to have them express how they’re doing it in a step-by-step process that will guarantee that they will find the term (and is more efficient than looking through every page).

2 Searching for Terms

1. The students are divided into groups of 3
2. Every group is given a phonebook.
3. They are asked to search for a few terms (something general, like “Restaurants” rather than McDonalds) and record the page numbers.
4. The fastest group is awarded candy.

3 Analysis

1. We ask the fastest group to explain how they searched for terms.
2. The explanations are usually a variant of estimating where the first letter of the word is in the book and doing the same approximation recursively for each letter.

3. Analyze the time it would take in the worst case of a brute force search.

The students will all know that doing a brute force search will take a “long” time, but we will actually figure out how long. Given that it takes a second to compare the term on the top of the page with the term you’re searching for and there are a total of n pages, we compute that it would take n seconds to find the page you’re looking for in the worst case.

They all know that they do better than that, but I point out that with the brute force search, it takes just as long if the pages weren’t sorted. So, I now ask them to write an algorithm (or step-by-step process) of how to they found the term they were looking for. The requirements were that by following it you would guarantee that they would find it and that it was efficient.

4 Review

Once they have finished writing it up, we review the groups’ algorithms and analyze (informally) how long it would take to find the page. Most of the algorithms are usually linear or hard to measure (since they do something similar to a phonebook search). It is then that we (the instructors), generalize how they do it by applying binary search, which follows as such

1. Go to the middle of the book
2. Is the term the term you’re looking for there?
3. If yes, you’re done
4. if not go the the middle of where you are and which half of the book we know the term has to be in.
5. Repeat (excluding first step) until you find the term or determine it’s not there.

Then, using the chalkboard, we illustrate that **half** of the answers are eliminated every time we do this. With some hints, they should be able to figure out that it takes $\log n$ time to find the search term. Incidentally, when most people do a search on a phonebook, they do what is called a phonebook search, which under certain assumptions, runs in $O(\log \log n)$ time.