Quadratic Modeling Exercises

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1 Projectile Motion

1.1 Intro

1. Now we finally try to model projectile motion ourselves
2. We will try to simulate it
3. In some sense, you have all the tools you need: quadratics and solving systems of linear equations
4. However, you will find that you need more tools still to fully solve it
5. Also, even though this is a computer based simulated, you will find that figuring out the quadratic isn't as easy as it seems.
6. Oftentimes, you might wonder why we need to know how to solve all these math problems. From a pure mathematics perspective, they're intellectual curiosities. However, at the same time, solving something like projectile motion requires lots of math. Even what we're doing today isn't half as complicated as what engineers do in real life. Having said that, you will still get a good taste of why we need the quadratic formula, etc.
7. You guys may use your calculators to solve the quadratic equation

1.2 Play

Let them play with it for 10 minutes

At the end, ask how many times (on average) did they have to shoot the missile to get it in. Also ask how far they got (highest level).

1.3 Equation Derivation

First, we open up the program on `http://mathplayground.com/ProjectTRIG/ProjectTRIGPreloader.html`

Then we have a sample up so that we can work on the equation.

The equation they will recite and get wrong is:

$$ax^2 + bx + c = y \tag{1}$$

We now state that acceleration due to gravity is $-9.8m/s^2$ but for this equation is $\frac{-9.8}{2}m/s^2$

We also take a look at the height of the cannon, which is going to our y-intercept. So $c = h_0$

So our equation looks like

$$\frac{-9.8}{2}x^2 + bx + h_0 = y \tag{2}$$

- Are we done?
- How many points do we need to solve this equation? (Hint: we need at least 1 point since we want the missile to land at a particular spot).
- The answer is 1 point.
- Does that sound right, though?
- That means we there is exactly 1 equation, 1 parabola, 1 path that the missile can take to get to the destination.
- Show the students (by making the parabola go higher) that there are an infinite number of solutions to get to the destination.
- This implies that we have more degrees of freedom than 1.
- In fact, we have 2 because we can control the angle and velocity.
- So the equation is not quite right.

The reason it's not right is because in this entire equation, there is no sense of time. Velocity is the change of y with respect to time over x with respect to time.

Students should recall the unknown value, b, is the initial velocity. The initial velocity has 2 components, x and y.

So, the equation really is:

$$\frac{-9.8}{2}t^2 + bt + h_0 = y \tag{3}$$

b now stands for initial velocity of y.

What about x? The velocity of x does not change (gravity does not affect horizontal speed), so the equation for x is:

$$x = dt \tag{4}$$

Or,

$$\frac{x}{d} = t \tag{5}$$

Where d is the initial x velocity.
So, now we substitute t with $\frac{x}{d}$

$$\frac{-9.8}{2}\frac{x}{d}^2 + b\frac{x}{d} + h_0 = y \tag{6}$$

Now we have 2 unknowns, which means we have 2 degrees of freedom. That corresponds with the fact that we have 2 controls to play with: velocity and angle. How many points do we need to solve this equation? Answer: 2.

1.4 Finding the correct parameters

- Turn on the grid
- Get the inital height and destination coordinate.
- Find a coordinate midway between the start and the destination that doesn't hit the bar.
- Plug in those coordinate for the equation and put it in woflramalpha.com

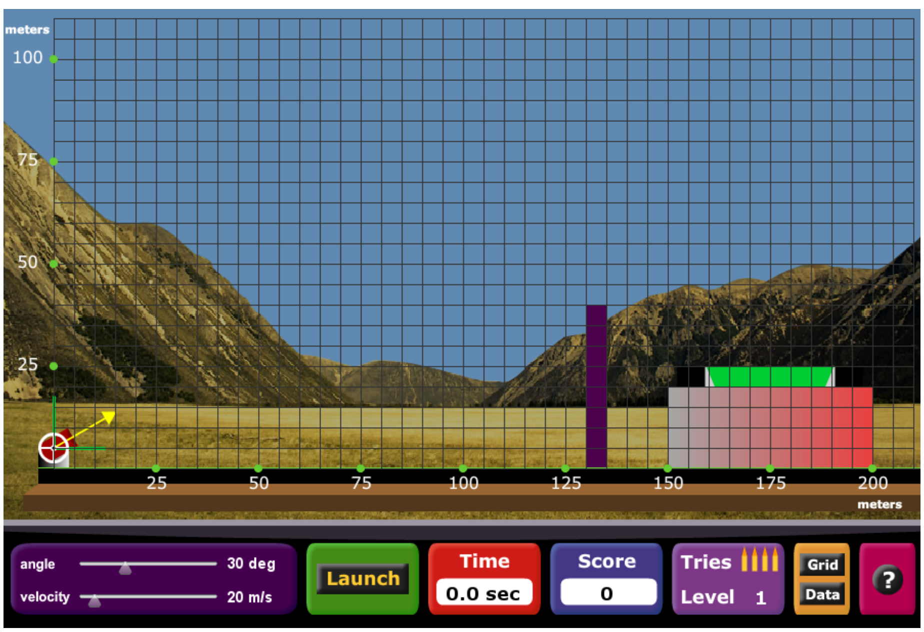

Figure 1: The projectile game. This example is what I'm basing my calculations off of. The target is (175,25). The initial height is 5. I want it to be at (100,75) as well

Plugging in these coordinates gives me:

$$\frac{-9.8}{2}\frac{175}{d}^2 + b\frac{175}{d} + 5 = 25 \tag{7}$$

and

$$\frac{-9.8}{2}\frac{100}{d}^2 + b\frac{100}{d} + 5 = 75 \tag{8}$$

To solve for them, I set them both to zero and equal to each other

$$\frac{-9.8}{2}\frac{175}{d}^2 + b\frac{175}{d} + 5 - 25 = 0 = \frac{-9.8}{2}\frac{100}{d}^2 + b\frac{100}{d} + 5 - 75 \tag{9}$$

which gives us
$b = 43.779$
and
$d = 20.71$
Can be computed using WolframAlpha `http://www.wolframalpha.com/input/?i=solve+(-9.8/2)*(175/d)^2+%2B+(175/d)*b++%2B+5+-25+%3D+0+%3D+(-9.8/2)*(150/d)^2+%2B+(150/d)*b++%2B+5+-65`

But we're not done (not quite). We know the individual components of velocity (x and y) but not the velocity and angle.

To compute it, notice that the x and y components make a triangle.

The hypotenuse is the initial velocity (use Pythagoreas' Theorem to find it).

To find the angle: just take the arc tangent of the ratio $\frac{b}{d}$. They don't know this yet, so mentioning it as such is important.

So, we get

$$b^2 + d^2 = v^2 \tag{10}$$

$$\sqrt{b^2 + d^2} = v \tag{11}$$

$$\sqrt{43.779^2 + 20.71^2} = 48.304 \tag{12}$$

Link: `http://www.wolframalpha.com/input/?i=sqrt(43.779^2+%2B+20.71^2)`

For the angle

$$atan(\frac{b}{d}) = angle \tag{13}$$

$$atan(\frac{43.779}{20.7063}) = 64.69 \tag{14}$$

Link: `http://www.wolframalpha.com/input/?i=arctan(43.779/20.7063)`

Rounding off, we get $velocity = 48$ and $angle = 65$

1.5 Final notes

Changing the velocity and angle even slightly can throw it off by a lot. Plus, even if there were not rounding errors, the missile might theoretically land in the green zone, but it could be that the black bar is in the way, so it fails.