

Quadratic Equation Exercises

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1 Reasoning

In the last few weeks, Michael and I have noticed that your reasoning skills need some developing. Most of you could probably do all right on an exam; but only if there are no “hard” questions.

These so-called “hard” questions are hard because they cannot be solved by some plug-and-chug method. You have to understand what you’re trying to solve. That would demonstrate that you have a good grasp of what you’re doing. Furthermore, if you make this class a class of memorization and not understanding, you will come to hate mathematics like so many before you. Learning like that is just like learning a bunch of random symbols and procedures that mean nothing. **No one** likes learning meaningless things.

No matter how many examples and presentations I give, you will not understand how to solve problems and—just as important—how to ask good questions unless you really understand what you’re dealing with. For this reason, this set of exercises have been created.

If, for any reason, you’re not sure how to do the problem or you don’t understand the question, feel free to e-mail Michael and I.

My e-mail is qwvako@email.arizona.edu. Just remember that teaching isn’t my my main “job”, so I might take a while to respond. But I’m more than happy to help.

2 Equation and Properties

Every equality is, in fact, a **truth statement** or a **proposition**. That is, it is a statement that you can either show it to be true or false. For example, you know that $3 \neq 2$ by definition of numbers (to really prove it, you need to learn more fundamental mathematics like the axioms of Peano arithmetic). Similarly, you know that $99 = 99$ is a true statement (every number is equal to itself).

Does this sound boring? Maybe. But that is at the heart of what happens in every mathematical equality. When you are comparing 2 numbers, the statement can only be true or false, not both. However, when you use variables, like

in an equation for a line, like

$$y = x + 5 \tag{1}$$

the truth of this statement is now ambiguous. Another way to say it is simply that whether this statement is true or false depends. But what does it depend on?

2.1

Given the equation,

$$y = x^2 + 1 \tag{2}$$

Answer the two questions:

1. When is this statement true? Give an example.
2. When is this statement false? Give an example.
3. When is this statement true in general? When is it false in general?

Hint: play with the values of x and y to get your answers. For the third question, Do **not** give me specific points as answers. Give me a general idea of when it's true or false.

2.2

A computer only understands the mathematical language. Whatever you tell it has to be precise and clear, otherwise it won't understand what to do.

Let's say you want to draw a line to split the screen horizontally (so you can play Halo cooperatively). If you know what the screen width and height are, w and h , how would you tell a computer which pixels to draw to split to screen?

Hint: It's an equation.

2.3

Given a system of equations

$$y = 3x + 6 \tag{3}$$

$$y = \frac{1}{2}x + 3 \tag{4}$$

1. What do the equations represent (graphically)?
2. If I set them equal to one another, when is the statement true?
3. Give me an example of when the statement (setting the two equations equal to each other) is not true? Explain what it means.

2.4

Given the equations

$$y = x^2 \tag{5}$$

$$y = |x^2| \tag{6}$$

Answer the following questions:

1. If x is a positive integer, will the output be the same for both equations?
2. If x is a negative integer, will the output be the same for both equations?
3. For the first equation, assuming x can be a complex number, can y ever have a negative value?
4. If x is a complex integer, will the output be the same for both equations?
5. For the second equation, assuming x can be a complex number, can y ever have a value of -1?

2.5 Stump Qiyam

I want you to come up with a legitimate question that you think can stump me.

I don't want you to go online and look for problems that no one has solved yet. Instead, I want you to think of something that confuses you. Think about how to put that into a form of a question and write it down.

If you think you understand the material pretty well, look over the material we've covered and try to come up with a combination of ideas that are hard to solve/understand. For example, a question might be something like

1. Evaluate $(2^i + 1)^3$
2. Why is $\frac{1}{\infty} = 0$?

Obviously, don't pick the questions I already listed (but you can ask them in class if you still want to know).