

Knights and Knaves

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1 Logic Problems

Mathematics is inextricably tied to logic. At the heart of every mathematical proof is the requirement that it must logically follow. Today, we will practice logic with a few problems.

2 Knights and Knaves

Knights and Knaves is a logic problem that involves (surprisingly!) people who live on island. On this island, there are only knights and knaves. Now, knights are people who always tell the truth and knaves always lie. The task is to logically deduce a fact (usually on the identity of the knight or knave). This is usually done by piecing together information from what the inhabitants say.



Figure 1: In knights and knaves problems, knights are people who always tell the truth while knaves always lie

To give you an idea of how this works, we'll go through an example.

3 Example Problem

3.1 Problem

There are two people, John and Jack.

John says: We are both knaves.

Who is what?

3.2 Reasoning

John and Jack can't both be knaves. If John and Jack are both knaves, then John would be telling the truth, which can't be because knaves can't tell the truth.

John can't be a knight. If he were a knight, he is now lying and saying that he's a knave, and knights do not lie.

John can be a knave and Jack can be a knight. This now makes John's statement false which are what knaves say.

John has to be a knave and Jack has to be a knight. We've exhausted all possible combinations and the only one that could be true is this one. Therefore, it is the correct deduction.

To solve the other problems, you have to do similar reasoning.

4 Problems

On this island, you meet two people: John and Jack. Solve the following problems and **write out your reasoning**. *Note: it is not always the case that there is a knight and a knave. It might be the case that both are knights or both are knaves.*

4.1

John: We are the same kind.

Jack: We are of different kinds.

Who is what?

4.2

John: Jack is a knave.

Jack: Neither John nor I are knaves

Who is what?

4.3

John and Jack are standing at a fork in the road. You know that one of them is a knight and the other a knave, but you don't know which. You also know that one road leads to Death, and the other leads to Freedom. By asking one yes/no question, can you determine the road to Freedom?