

A Lesson on Functions

Qiyam Tung

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1 What is a function and why do we study them?

Imagine that you're using Adobe Photoshop. Photoshop allows us to manipulate images in many ways, like rotating an image so that it looks correct (1). Intuitively, you all know how to rotate 90 degrees. But would you know how tell a computer how to rotate an image? Specifically, if given a single point or pixel, how would you rotate 90 degrees? The method of rotating that point is called *function*. If we were to describe it more accurately, the rotation function takes a pixel and outputs a pixel that has been rotated 90 degrees. Similarly, there is a function that describes how to shift and enlarge an image.



Figure 1: We can use computer software to rotate images so that they are more pleasing to looking at. A rotation of an image is a kind of function. Image from a Photoshop tutorial online.

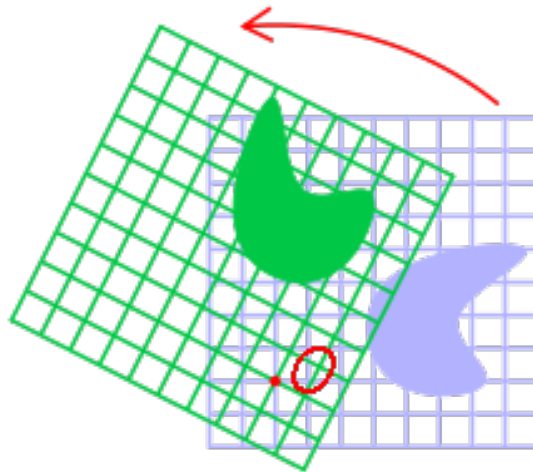


Figure 2: How would you mathematically describe a rotation? If you knew how to do that, you know how Photoshop rotates images. Image from Wikipedia.org.

Everything a computer does can be described by a function. Naturally, you might wonder whether there are different *kinds* of functions? But how can you ask that if you don't know what a function is in the first place?

2 Reading

First, you need to read <http://en.wikibooks.org/wiki/Algebra/Functions>. Read up to and *including* the section on *Composite function*.

I would recommend doing the Practice Problems, but you do not have to. However, if you are struggling to get the right answers in the following sections, use them for practice as you can look up the answers online.

3 Function Definition

In mathematics, if unsure of how to answer a question, always refer to the definition. This is rather like how referees determine whether a score was made or not: by referring to the rule book. Unfortunately, the modern definition of a function requires you to have a bit of background in set theory. Instead, we'll use the Wikibook definition, but add additional info. **A function is process that takes an input (or multiple inputs) but only creates one output.**

Here are a few examples of functions in real life:

- Juice blender. Its input are fruits and output is juice.
- Keyboard: input is an electrical signal from the keys and its output is a character on the screen.
- Grades: input hours spent studying, output is grade percentage on test.

Let's work with the last example.

We could make a very simple function and call it *grade*

$$grade(hours) = 10 * hours \quad (1)$$

Now, this function says that if I study 0 hours, I get a grade of 0%. I must study for 10 hours if I want to get a grade for 100% and 9 hours for 90%, etc. Note that is not a very good function of how many hours you have to put in. That is, moving from a 10% to 20% grade takes the same number hours of study to move from 90% to 100%. In reality, it takes much more time and effort to move from a 90% to 100% than 10% to 20%. This function was just used to illustrate how you can make your own functions.

Now, it's a pain to have to write out *grade* and *hours* all the time, so we can abbreviate them and call them *g* and *h*, respectively. Now our equation looks like

$$g(h) = 10 * h \quad (2)$$

Look familiar?

4 Questions

Translate the following english sentences into mathematical functions. Use the most appropriate letters.

1. My age is a function of my younger brother's age. He is two years younger than me.
2. When Sunnyside and Desert View go against one another, Sunnyside wins with twice the number of points Desert View and an additional 3 points. Sunnyside's score is a function of Desert View's score.
3. My salary is a function of how many hours I work. However, no matter how many hours I work, I always earn 50000 dollars.

5 Calculus Only

1. Come up with a more accurate function that describes a student's grade as a function of hours studied than the one I came up with. Specifically, it has to show that you spend more time studying to move from a 90% to a 100% than from a 10% to 20%. Also, I must be able to get any grade. That is, if the grade is known but the hours studied is unknown, it must be possible to use your equation to figure out how many hours was studied.