

A Lesson on Functions

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1 More Reading

It is often helpful to learn from two different sources. Once again, read <http://en.wikibooks.org/wiki/Calculus/Functions> from the beginning until the end of “Composition of Functions.”

Note that this reading is more precise and is intended for Calculus students. However, it is not so deep so that it will confuse Algebra 2 students.

2 Rewriting functions

As I demonstrated in class, every operation you know (addition, subtraction, multiplication, division, etc.) can be written in functional notation.

For example, adding one to a number can be defined as:

$$\text{addone}(x) = x + 1 \tag{1}$$

So, adding 1 to 2 is $\text{addone}(2) = 2 + 1 = 3$. But you can define addition in general by simply giving the function two inputs.

$$\text{add}(x, y) = x + y \tag{2}$$

When you have more than one operation/function in an expression, like $3x + 1$, you will have to nest functions. For example, the previous expression can be written as $\text{add}(\text{multiply}(3, x), 1)$.

2.1 All students

For the expressions, rewrite them so that they're in functional notation. For example, $y = 2x$ must be written as $\text{multiply}(2, x) = 2x$.

1. $y = x^2$
2. $y = x * y + 3$
3. $y = (7x - y) * 5$
4. $y = \frac{7x+5}{x^2-2y}$
5. $y = 1$
6. $y = \log_y(x)$
7. $y = \log_2 x$
8. $y = \sin(2x)$

2.2 Calculus Only

Now that you know how to write any expression in functional notation, you should be able to justify why the derivative of the given expression is the way it is.

For example,

$$\frac{d}{dx} \sin(x^2) \quad (3)$$

The chain rule tells us that

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x) \quad (4)$$

So, to solve the problem, we need to rewrite the function so that we can use the chain rule. We can say that $g(x) = x^2$ and $f(x) = \sin(x)$. This implies that $f(g(x)) = \sin(g(x)) = \sin(x^2)$. So, to take the derivative I just need to take the derivative of each function. $f'(g(x)) = \frac{d}{dx} \sin(x^2) = \cos(x^2)$ and what $g'(x) = \frac{d}{dx} x^2 = 2x$.

Therefore,

$$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot 2x \quad (5)$$

Problems.

Take the derivative of each expression with respect to x . Use functional notation to justify your solution. For some of these problems you will not know how to take the derivative. If you don't know how to do it, explain why.

1. $y = \tan(x)$
2. $y = \tan(3x)$
3. $y = \sin(\cos(x))$
4. $y = x^{\sin(x)}$
5. $y = \ln(\ln(x))$