Making Formulas to Solve Linear Systems

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1 Creating a formula

Formulae are plug-and-chug methods of finding an unknown quantity. For example, you know the formula to calculate the slope of a line:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \tag{1}$$

However, how do you come up with formulae? Wouldn't it be nice if there were a method easier than what you're currently doing to solve systems of linear equations?

For example, let's say we have two equations

$$2x + y = 3 \tag{2}$$

$$3x + y = 5 \tag{3}$$

You know how to solve this. But let's say we see lots of systems of equations, like $\frac{15}{2}x+y = 13$, $\frac{13}{7}x + y = 10$, etc. What similarites do all these equations share? All the y-coefficients are one.

Perhaps we can make life simpler if we found a faster method of solving systems of linear equations when y has a coefficient of one.

To generalize, first let's assume the coefficient of x can be anything, call it A. Let's also call the constant C. So now we have

$$Ax + y = C \tag{4}$$

But wait, we need two equations to solve for this right? We can't call both x coefficients A, otherwise that means they're the same number. So, let's modify it so that the x coefficients of the first equation are called A_1 and the second equation's is A_2. Same thing with C.

$$A_1x + y = C_1 \tag{5}$$

$$A_2x + y = C_2 \tag{6}$$

How would we solve for this equation? Well, cancel the y's first. Subtract 5 from 6.

$$(A_2 - A_1)x = C_2 - C_1 \tag{7}$$

$$x = \frac{C_2 - C_1}{A_2 - A_1} \tag{8}$$

Putting this back in for x, we get

$$A_2x + y = C_2 \tag{9}$$

$$A_2 \cdot \frac{C_2 - C_1}{A_2 - A_1} + y = C_2 \tag{10}$$

$$y = C_2 - A_2 \cdot \frac{C_2 - C_1}{A_2 - A_1} \tag{11}$$

Does this really work? Let's try it.

$$2x + y = 3 \tag{12}$$

$$3x + y = 5 \tag{13}$$

$A_1 = 2,\ A_2 = 3,\ C_1 = 3,\ C_2 = 5.$

$$y = 5 - 3 \cdot \frac{5-3}{3-2} \tag{14}$$

$$y = 5 - 6 \tag{15}$$

$$y = -1 \tag{16}$$

$$x = \frac{5-3}{3-2} \tag{17}$$

$$x = \frac{2}{1} \tag{18}$$

$$x = 2 \tag{19}$$

So, $x = 2$ and $y = -1$. Plugging those numbers back in confirms it.
Do the same for

$$x + B_1 y = C_1 \tag{20}$$

$$x + B_2 y = C_2 \tag{21}$$