

Geometric Probability

When trying to determine the likelihood of an event, we are trying to determine the percent of the time that the event would occur if the experiment were repeated an infinite number of times. Since we do not like to do calculations with percents, we look for numbers between 0 and 1.

Conceptually, we calculate probabilities by looking at ratios. In the numerator we write the total "number" of desired outcomes and in the denominator we write all outcomes, desired or undesired.

For example, let's say a bag contains 4 red marbles and 6 blue marbles and I want to calculate the probability of drawing a red marble at random. The solution is

$$\frac{\textit{desired}}{\textit{total}} = \frac{\textit{red}}{\textit{total}} = \frac{4}{10}$$

So there is a 40% chance of drawing a red marble.

Some probability problems are geometric in nature. What this means is that our desired outcome space is in fact some area called the feasible region. To find the probability of this outcome we find the ratio

$$\frac{\textit{feasible region}}{\textit{sample space}}$$

A classic example of this type of problem is a dart board. Let's say that we have a dart board that has a 10 in radius. Furthermore, the bulls-eye is exactly in the center and has a 1 in diameter. If I throw a dart randomly and it hits the board, what is the probability that it hits the bulls-eye?

Here is another classic problem. Imagine a table that is 2 feet square and a circle is inscribed on the surface of the table. The game is to throw a coin on the table and have the entire coin lie within the circle with no part touching the region outside of the circle. If the coin has a diameter of 1 in, what is the probability of the coin lying within the circle? (Assume all coins land on the table)

Some more interesting problems can be turned into geometric probability problems. We can use a coordinate system to set them up. Here are some examples.

- Suppose two numbers, x and y , are generated at random, where $0 < x < 3$ and $0 < y < 6$. What is the probability that the sum is less than or equal to 2?
- Suppose two numbers, x and y , are generated at random, where $0 < x < 1$ and $0 < y < 1$. What is the probability that the quotient $\frac{y}{x}$ is between 2 and 3?
- Suppose two numbers, x and y , are generated at random, where $0 < x < 1$ and $0 < y < 1$. What is the probability that the product of the two numbers is less than $\frac{1}{2}$?
- Suppose two numbers, x and y , are generated at random, where $0 < x < 4$ and $0 < y < 4$. What's the probability that the sum of the numbers exceeds the product?

Using the techniques in the previous problems, we can actually solve some interesting applications.

Two friends, Lucie and Ann are shopping at the mall. They agree to split up for a time and then meet for lunch. They plan to meet in front of the restaurant between 12:00 noon and 1:00 PM. The one who arrives first agrees to wait 15 minutes for the other to arrive. After 15 minutes, that person will leave and continue shopping. What is the probability that they will meet if each of the girls arrives at any time between 12:00 noon and 1:00 PM?

The trick here is to let Lucie's arrival time be x , and Ann's arrival time be y . If they arrive at the same time then $x = y$. If Lucie arrives first, then Ann arriving in time means $y \leq x + 15$. Similarly, if Ann arrives first, then Lucie arriving in time means $x \leq y + 15$.