Μσηδαγ Ματη Μαηια

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Fermat's Last Theorem

Pierre de Fermat was an amateur French mathematician who lived in the 17^{th} century. He was an amateur mathematician only in so far as he was a lawyer by trade, as his mathematical discoveries rival those of the greatest mathematicians of all time, including Carl Gauss. Like Gauss, Fermat was particularly interested in number theory and geometry, although his genius was not limited to these subjects. In fact, his correspondence with Blaise Pascal laid the groundwork for the theory of probability. While Fermat made significant mathematical contributions while he was alive, his greatest contribution would come posthumously.

Pierre de Fermat

After Fermat's death in 1665, his son Clement-Samuel spent five years collecting his father's note and letters, and examining his jottings in the margins in his copy of the book *Arithemtica* (A famous greek text on mathematics written by Diophantus). One day, he came across the following note scribbled in the margin, a note that would haunt mathematicians for centuries,

> It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. *I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.*

What Fermat was saying is that for any whole number $n > 2$, there are no non-trivial whole number solutions to the equation $x^n + y^n = z^n$ (non-trivial means $x, y, z \neq 0$). This became known as **Fermat's Last Theorem**.

For the next 330 years some of the greatest minds of mathematics and science struggled to prove Fermat's Last Theorem without success. In fact, many mathematicians had decided to put Fermat's Last Theorem to one side because they considered it impossible. For example, the great mathematician David Hilbert was asked why he did not attempt a proof of the Last Theorem, and he replied, "Before beginning I should have to put in three years of intensive study, and I havent that much time to squander on a probable failure." Despite the fact that no proof was found, the different means of attacking Fermat's Last Theorem generated an explosion of new mathematical ideas. In fact, the foundations of what is known as algebraic number theory came from search for a proof.

Andrew Wiles - taken minutes after he completed his lecture, where he presented his proof of Fermat's Last Theorem for the first time.

Andrew Wiles first became aware of the Last Theorem when he was ten years old. On his way home from school he popped into the Milton Road library and began reading *The Last Problem* by Eric Temple Bell. From that moment on, he dedicated his life to finding a proof, even though this was something that had eluded the greatest brains on the planet for three centuries.

He completed his PhD in mathematics and eventually became a professor at Princeton University. His research was in number theory, but was not aimed at Fermat's Last Theorem. However, in the 1980s, two mathematicians named Ken Ribet and Gerhard Frey built a bridge between Fermat's Last Theorem and mainstream mathematics. Inspired by this new discovery, Wiles worked in complete secrecy for the next seven years, formulating the proof of the century.

At a mathematical conference in 1993, rumors began to fly that Wiles had solved the problem. Before a standing room only audience, Wiles filled blackboard after blackboard with his proof. Finally, he wrote

Fermats famous equation, and famously said "I think Ill stop here." Wiles received a boisterous standing ovation, something unheard of in mathematical lectures.

In 1995, Wiles's proof was officially accepted by the mathematical community and published in the prestigious *Annals of Mathematics*. We now know that Fermat's Last Theorem is true, but one question remains. What was Fermat's original proof? Wiless proof is far too complicated to be the same as Fermat's, so some people continue to search for the original proof if, indeed, such a proof exists it could be that Fermat made a mistake and never had his own proof.

The following quote by Andrew Wiles, which draws an analogy between doing mathematics and exploring a dark mansion:

> "You enter the first room of the mansion and its completely dark. You stumble around bumping into the furniture but gradually you learn where each piece of furniture is. Finally, after six months or so, you find the light switch, you turn it on, and suddenly it's all illuminated. You can see exactly where you were. Then you move into the next room and spend another six months in the dark. So each of these breakthroughs, while sometimes they're momentary, sometimes over a period of a day or two, they are the culmination of, and couldn't exist without, the many months of stumbling around in the dark that precede them."

If you would like to learn more about Fermat's Last Theorem check out the excellent book *Fermat's Enigma* by Simon Singh and the PBS Nova film *The Proof*.

Problems of the Week

Below are three problems of the week. The problem with three stars is considered the most difficult, while the problem with one star is considered to be the least. Students that correctly solve a problem will have their names printed in the next Monday Math Mania. Have fun and good luck!

(★) You are given eight blocks: two red, two blue, two green, and two yellow. You are asked to stack the blocks according to the following four rules:

1. Just one block must lie between the two red blocks.
2. Two blocks must lie between the pair of blue blocks.
3. Three blocks must separate the pair of green blocks.
4. Four blocks must separate the pair of yellow blocks.

Can you figure out how to do it?

(★★) You're email password is 9 characters long. The first five characters must be upper or lowercase english letters (a,A,b,B,c,C,...), and each of the last four characters must be a whole number between 0 and 9. Suppose a computer hacker wrote a computer program that could try to log into your email account with a different password every .5 seconds, how long would it take before the hacker was able to try every possible password? Express your answer in years.

(★★★) There are 3 boxes, each containing two coins. One box contains two gold coins, one contains one gold coin and one silver coin, and one contains two silver coins. If you randomly select a box and pull out a gold coin, what is the probability that the other coin in the box is also gold?

Submit your solutions to Mr. Lafferty or Mrs. Gould by 12:40 on Friday, or email your solution to contact@stmichaelsmathmania.net by 11:59pm Friday.