

$$\begin{aligned} 1 \text{ googol} &= 10^{100} \\ 1 \text{ googolplex} &= 10^{\text{googol}} = 10^{10^{100}} \end{aligned}$$


- If you thoroughly shuffle an ordinary deck of 52 playing cards, chances are that the resulting arrangement of cards has never existed before!

The 52 card deck is believed to be invented approximately 300 years ago. Suppose that everyone on Earth today (approximately 7 billion people) has played cards every day for the past 300 years, and each time shuffled the deck 1000 times (I'm sure you'll agree this is an extremely generous estimate of the number of possible shuffles). Then we have

$$\underbrace{7,000,000,000}_{\text{people}} \times \underbrace{300}_{\text{years}} \times \underbrace{365}_{\text{days/years}} \times \underbrace{1000}_{\text{shuffles/day}} = 7.6 \times 10^{17}$$

while the number of possible shuffles is

$$52! = 52 \cdot 51 \cdot 50 \cdots 2 \cdot 1 = 8.1 \times 10^{67}.$$

So, the chance of shuffling a deck of cards an getting the same order as some previously shuffled deck is

[illegible]

- The current world record for remembered digits of  $\pi$  is 67,890!

Lu Chao, a 24-year-old graduate student from China recited 67,890 digits of  $\pi$  without an error. It took him 24 hours and 4 minutes. He actually memorized 100,000 digits of  $\pi$  but made a mistake on the 67,891<sup>st</sup> digit.

- Solving Rubik's Cube



- **How many people do you need to choose at random in order to have a 50% chance that two have the same birthday? Just 23!**

Suppose you choose 23 people at random. The probability that NONE of them share a birthday is

$$\frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdots \frac{343}{365} = 0.49270276.$$

So the probability that at least two of them share a birthday is 1 minus the above probability, that is,

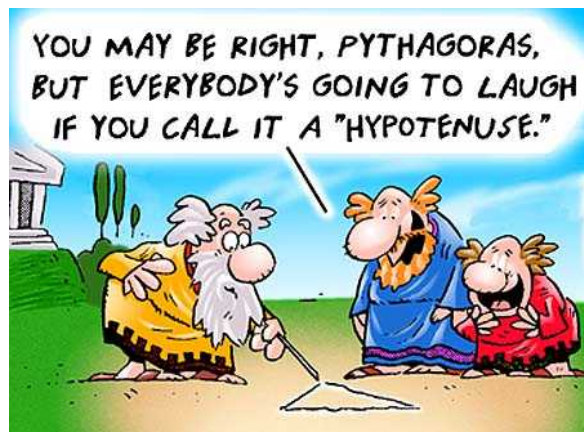
$$1 - \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdots \frac{343}{365} = 0.507297.$$

or, 50.7297%! In fact, using the same argument as above, one can show that the you only need 57 randomly selected people to have a 99% probability that two of them share a birthday!

- **Overboard!**

Pythagoras (580-500, BC), the guy that gave us the famous theorem about right triangles, had his own school. The dictum of the Pythagorean school was "All Is Number". In fact, the Pythagoreans were not unlike modern day numerologists and attributed certain mystical properties to numbers. Central to the beliefs of the Pythagoreans was that every number is rational, that is, every number can be expressed as a fraction in which the numerator and denominator are integers.

The discovery of the first irrational number was greeted with less than enthusiasm. While the Pythagoreans were at sea, one of their members, Hippasus of Metapontum, produced an element,  $\sqrt{2}$ , and showed that it cannot be expressed as a fraction in which the numerator and denominator are integers. He was thrown overboard.



- **How fast is Santa's Sleigh?**

Let's assume that Santa only visits those who are children in the eyes of the law, that is, those under the age of 18. There are roughly 2 billion such individuals in the world. However, Santa started his annual activities long before diversity and equal opportunity became issues, and as a result he doesn't handle Muslim, Hindu, Jewish and Buddhist children. That reduces his workload significantly to a mere 15% of the total, namely 378 million. Now, according to the most recent census data, the average size of a family in the world is 3.5 children per household. Thus, Santa has to visit 108,000,000 individual homes. (Of course, as everyone knows, Santa only visits good children, but

we can surely assume that, on an average, at least one child of the 3.5 in each home meets that criterion.)

That's quite a challenge. However, by traveling east to west, Santa can take advantage of the different time zones, and that gives him 24 hours. Santa can complete the job if he averages 1,250 household visits per second. In other words, for each Christian household with at least one good child, Santa has 1/1250th of a second to park his sleigh, dismount, slide down the chimney, fill the stockings, distribute the remaining presents under the tree, consume the cookies and milk that have been left out for him, climb back up the chimney, get back onto the sleigh, and move on to the next house. To keep the math simple, let's assume that these 108 million stops are evenly distributed around the earth. That means Santa is faced with a mean distance between households of around 0.75 miles, and the total distance Santa must travel is just over 75 million miles. Hence Santa's sleigh must be moving at 650 miles per second – 3,000 times the speed of sound. A typical reindeer can run at most 15 miles per hour. That's quite a feat Santa performs each year!

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## Problems of the Break

Below are several problems for the break. The problem with three stars is considered the most difficult, while the problem with one star is considered to be the least. Students that correctly solve a problem will have their names printed in the next Monday Math Mania. Have fun and good luck!

(★) Suppose there are three boxes, each containing 2 balls. In one box there are two white balls, in another two black balls, and in the third, one ball is black, the other is white. The boxes have been labeled to indicate their contents, however, whoever did the job got **all** labels wrong. Can you figure out what is in each box by selecting 1 box and blindly picking a ball out of it? If so, why? If not, why not?

(★) Is Pinnochio telling the truth? If so, why? If not, why not?



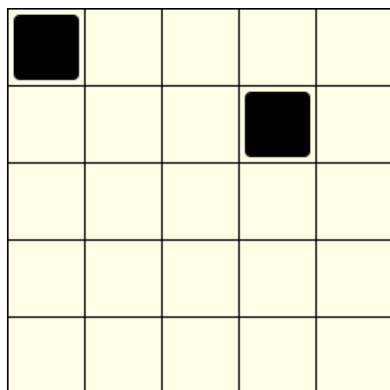
(★★) Consider the Target logo to the right. Suppose the center red circle has radius 1, and each ring around the center circle has width 1. If a blindfolded person throws a dart and hits the target logo, which is more likely:

- Your dart hit the red circle in the center.
- Your dart hit the white ring.
- Your dart hit the outer red ring.
- Your dart landed in the center red circle or white ring.

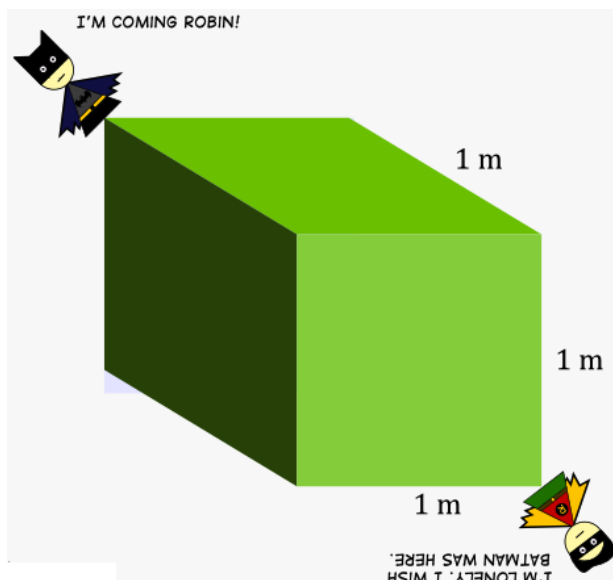


Be sure to give a clear explanation of why your answer is correct.

(★★) You are given 8 coins, 7 of which are the same weight and 1 which is heavier than the rest. Using a balance, what is the minimum number of weightings required to determine which coin is the heavier one? Explain how it can be done.



(★★) A few cells on a square grid are blocked. The task is to draw a path that passes through each blank square exactly once. The path can only change direction when it hits a dark square, a square that contains part of the path so far, or the edge of the grid.



(★★★) Batman and Robin were captured by the Riddler and are now trapped on the surface of the Riddler's cube. Batman is at one corner of the cube, while Robin is at the opposite corner. If Batman is confined to the surface of the cube, what is the length of the **shortest path** to Robin? Explain how you got your answer.

(★★★) If  $x^2 + y^2 = 15$  and  $xy = 5$ , what is  $x + y$ ? Explain how you got your answer.

(★★★) Five pirates (of different ages) have a treasure of 100 gold coins. On the ship, they decide to split the coins using the following scheme:

- The oldest pirate proposes how to share the coins, and all pirates remaining will vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way. Otherwise, the pirate proposing the scheme will be thrown overboard, and the process is repeated with the pirates that remain.

Assuming that all five pirates are intelligent, rational, greedy, and do not wish to die, what should the oldest pirate propose to a) survive and b) maximize his profit? Explain your answer.



Solutions to each week's problems will be posted at [www.stmichaelsmathmania.net](http://www.stmichaelsmathmania.net) after a successful solution has been given by a student.

Submit your solutions to Mr. Lafferty or Mrs. Gould by 12:40 on Friday, or email your solution to [contact@stmichaelsmathmania.net](mailto:contact@stmichaelsmathmania.net) by 11:59pm Friday.

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## Star Problem Solvers

The ★ problem from Nov. 1 was solved by: **Arica Christensen**

The ★★ problem from Nov. 1 was solved by: **Will Vail**

The ★★★ problem from Nov. 1 remains unsolved!

Last week's ★ problem was solved by: **Ben Arthur, Read Wilder, and Hunter Grimm**

Last week's ★★ problem remains unsolved!

Last week's ★★★ problem was solved by: **Read Wilder**