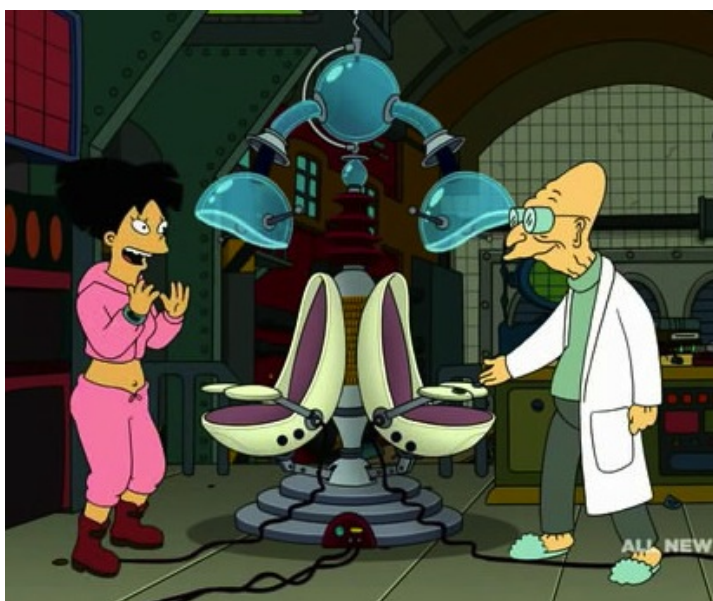


Some Serious Math Shows up in Futurama

Who said you can't learn anything from watching cartoons. Futurama is considered by some to be one of the smartest shows on TV, and the writers decided to take that to a whole new level in the August 10, 2010 episode titled "**The Prisoner of Benda.**"

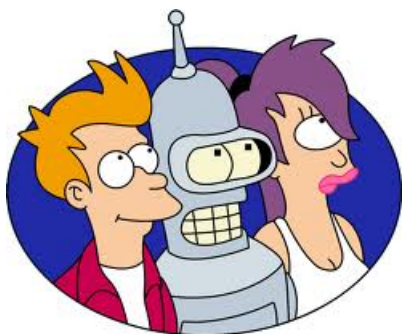
In the episode, Professor Farnsworth unveils a new invention of his which allows anyone to switch mind and body with another. It begins with Amy and Farnsworth innocently swapping bodies for personal reasons, but they soon learn that they aren't able to swap back. By the end of the episode, all the Planet Express crew, friends and acquaintances, and even robo-utensils have used the machine, and no one is in the correct body. To get everything back to normal calls for the assistance of the brilliant minds from the Globetrotter Home-world and some crafty mathematical equations.

Considering this is a cartoon and a comedy, writers of the episode could have easily just offered up a bunch of gibberish math consisting of random important-looking symbols. Instead, writer Ken Keeler, who also has a PhD in mathematics from Harvard, decided to not only craft a brand new mathematical theorem, but also PROVE it!



The gist of the theorem is as follows: suppose that k people have swapped bodies. Label these people so that the 1st person is in the 2nd person's body, the 2nd person is in the 3rd person's body, and so on, so that the last person (a.k.a. the k^{th} person) must be in the 1st person's body.

With two additional people (let's call them Xerxes and Yelena, or x and y for short) we can get everyone back to normal by using the following procedure: have the 1st body switch with y 's body, then have the k^{th} body switch with x 's body. Next, have the k^{th} body switch with y 's body, and then, in turn, have x 's body switch with the $(k-1)^{\text{th}}$ body, $(k-2)^{\text{th}}$ body, and so on, until x 's body has switched with the 1st body. At the end of this procedure, everyone will be back to normal except for x and y , who will be in different bodies. But since they haven't switched with each other yet, they can then switch back! If you would like to see the complete proof, check out the Futurama wiki at <http://theinfosphere.org>, and look for the episode "**The Prisoner of Benda.**"



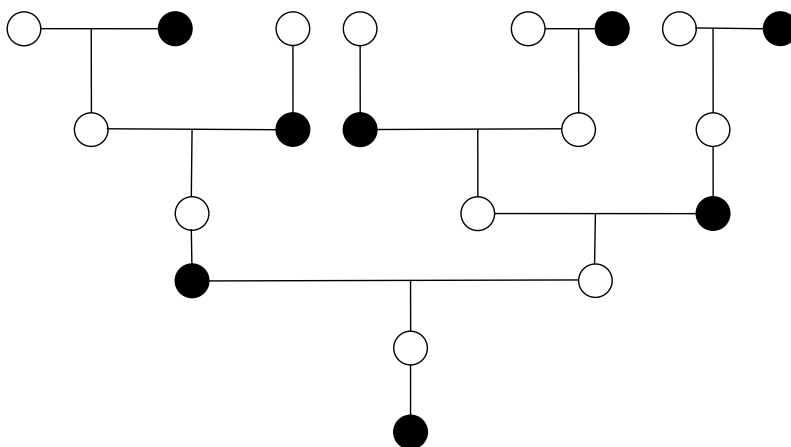
This theorem is in the mathematical area of group theory, specifically it has to do with something called the **symmetric group on n elements**. This gives rise to several questions: in a group of n people, what is the minimum number of swaps needed to return everyone to normal? What is the maximum number of times an individual's mind must swap in order for the group to return to normal? If we place caps on this maximum (say, because the cerebral immune response gets stronger with repeated use), what restrictions does this place on how much fiddling with the mind swapping device before people won't be able to return to their bodies?

If you are able to catch this episode when it airs again, do so.

While this result doesn't have far-reaching consequences, the fact that it is an original piece of work inspired by the plot of the episode is a great example of how critical thinking can be used to solve all types of problems.

Below are three problems of the week. The problem with three stars is considered the most difficult, while the problem with one star is considered to be the least. Students that correctly solve a problem will have their names printed in the next Monday Math Mania. Have fun and good luck!

(★★) Honey bees reproduce in a curious way. Male bees are born asexually, that is, they have only one parent (a female). Females are born of two parents (a male and a female). Shown below are six levels of the family tree of a typical male bee:



How many ancestors does a boy have six generations back? Ten generations back? Twenty? How many of those were female?

(★★★) Suppose we have N consecutive even integers, where N is also a positive even integer. If the sum of the first $N/2$ integers is 32 less than the sum of the last $N/2$ integers, and five times the smallest integer is 272 more than twice the sum of the largest two, what are the consecutive integers?

Solutions to each week's problems will be posted at www.stmichaelsmathmania.net after a successful solution has been given by a student.

Submit your solutions to Mr. Lafferty or Mrs. Gould by 12:40 on Friday, or email your solution to contact@stmichaelsmathmania.net by 11:59pm Friday.