Μσηδαγ Ματη Μαηια

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The Mathematics of Invisibility

J.K. Rowling may not have realized just how close Harry Potter's invisibility cloak was to becoming a reality when she introduced it in the first book of her best-selling series. Scientists, however, have made huge strides in the past few years in the rapidly developing field of cloaking. Ranked the number five breakthrough of the year by Science magazine in 2006, cloaking involves making an object invisible or undetectable to electromagnetic waves.

In October of 2006, David R. Smith, associate professor of electrical and computer engineering at Duke University, led a team that used a circular cloaking device to successfully bend microwaves around a copper disk as if the disk were invisible.

However, the mathematics behind this circular cloaking device had already been developed by Allan Greenleaf, a mathematician at the University of Rochester.

"We were working on improving the mathematics behind tumor detection," says Greenleaf. "In the final section to one paper, we spelled out a worst-case scenario where a tumor could be undetectable. We then wrote a couple of additional articles describing when this could happen. At the time, we didn't think further about it because it seemed extremely unlikely that any tumor would be covered with the necessary material to be hidden that way."

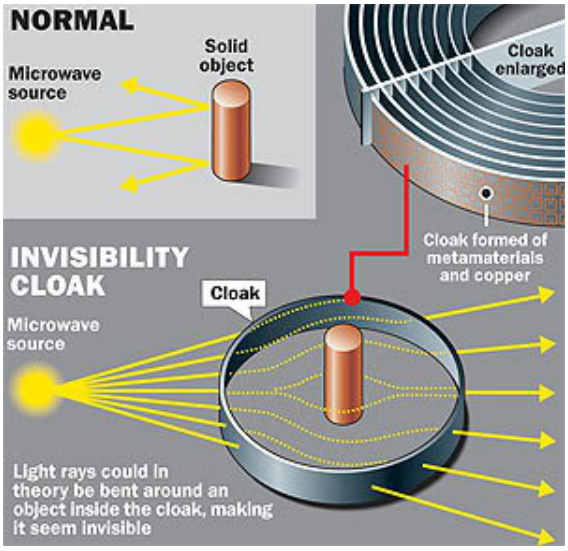
Once Greenleaf and his colleagues saw that their results could also be used to show how to "hide" an object, they decided to analyze and improve the proposed cloaking device, using the techniques they had developed in their earlier work. They knew that a crucial question would be: What was going on inside the cloaked region?

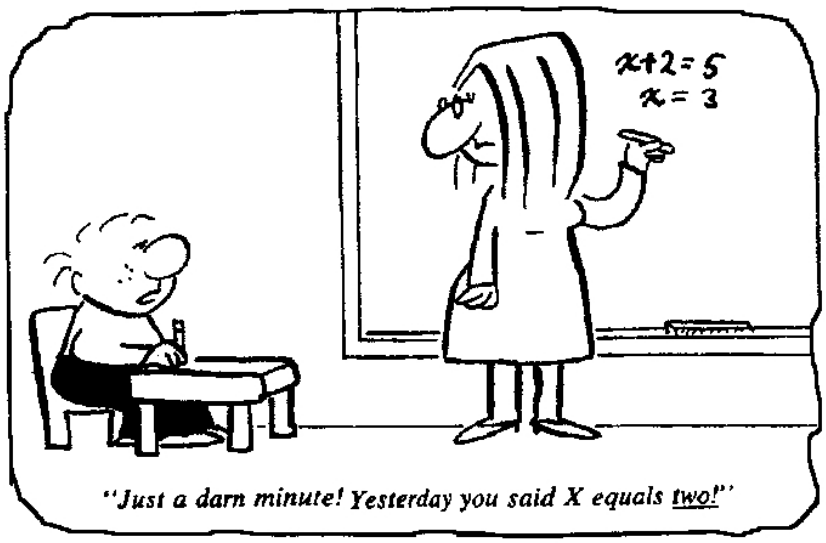
Greenleaf and his collaborators used sophisticated mathematics to understand what must be happening inside the cloaked region. They soon realized that anything that emitted electromagnetic waves such as a cell phone, a digital watch, or even a simple electric device like a flashlight, would cause the behavior of the cloaking device to go seriously awry.

Their analysis also revealed another surprise: a person trying to look out of the cloak would effectively be faced with a mirror in every direction. If you can imagine Harry Potter's own invisibility cloak working this way, and Harry turning on his flashlight to see, its light would shine right back at him, no matter where he pointed it!

"We should also keep in mind that, given the current technology, when we talk about invisibility, we're talking only about being invisible at just a narrow range of wavelengths," says Greenleaf. "For example, an object could be rendered invisible at just a specific wavelength of red; it would be visible in nearly every other color."

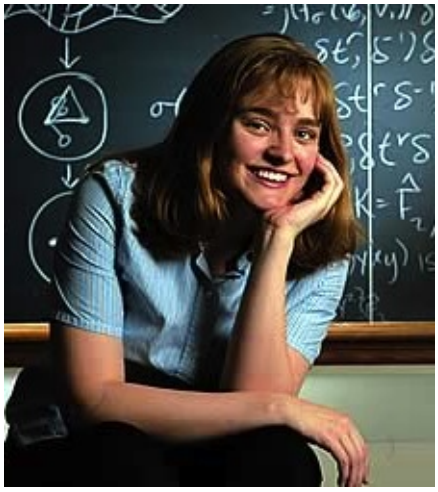
Smith's team at Duke is also working on improving their cloaking device. In December of 2006, Smith and Greenleaf met for the first time and talked about Greenleaf's new math. "Allan has been looking at the problem much more generally, and deriving the conditions for when true invisibility is or is not possible," says Smith. "We are very interested in what he and his colleagues come up with!"





Mathematician of the Week

Melanie M. Wood was born in 1981, in Indianapolis, where her mother began instructing her in mathematics at the age of three. However, it wasn't until middle school that Melanie's mathematical gifts emerged. In 1994, Melanie was asked to join the local MathCounts team where shocked everyone by finishing first in the state and 40^{th} in the nation. The following year she would finish 10^{th} in the nation.



While a high school student, Melanie Wood became the first female to make the U.S. International Math Olympiad Team. Shortly there after, Melanie was profiled as "The Girl Who Loved Math" by *Discover* magazine.

After high school Melanie attended Duke University. In 2002 she became the first American woman and second woman in the world to be named a Putnam Fellow (the top five or six scorers on the William Lowell Putnam Mathematical examination). The following year she was awarded the Morgan Prize, which is an annual award given to an undergraduate student in the United States for outstanding original mathematical research. Winning the prize was in large part due to a paper that she wrote and was published in the prestigious *Journal of Number Theory*.

Melanie just completed her PhD in Mathematics at Princeton University in 2009, and is now a professor of mathematics at Stanford University where she does research in number theory.

> "I think people often overlook the similarity between what it takes to be a mathematical genius and what it takes to be an artistic genius. Neither can be achieved by technical skill alone, yet certainly a great deal of technical skill is required for both. Both require creativity and the ability to think in ways that others haven't been able to imagine."
>
> - Melanie Wood

Problems of the Week

Below are three problems of the week. The problem with three stars is considered the most difficult, while the problem with one star is considered to be the least. Students that correctly solve a problem will have their names printed in the next Monday Math Mania. Have fun and good luck!

(★) Suppose we tied a rope snugly around the Earth's equator so there is no gap between the Earth's surface and the rope. Imagine lifting this very long rope off of the earth, cutting it somewhere and stitching it together with an additional foot of rope, and then putting this longer rope around the Earth's equator once more.

Since the rope is longer it will no longer fit snugly around the Earth's equator, so there will be a gap between the Earth and rope. How large is this gap?

(★★) Can you find a positive four-digit whole number n with the property that the last 4 digits of n^2 is the original number n?

(★★★) The factorial of a positive whole number n, which we write as $n!$, is given by $n! = n \cdot (n-1) \cdots 2 \cdot 1$. So for example, $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$. Suppose $n! = 1405006117752879898543142606244511569936384000000000$. What is n? (Hint: n is divisible by 3. By the way, how could we tell that?)

Solutions to each week's problems will be posted at www.stmichaelsmathmania.net after a successful solution has been given by a student.

Submit your solutions to Mr. Lafferty or Mrs. Gould by 12:40 on Friday, or email your solution to contact@stmichaelsmathmania.net by 11:59pm Friday.