

Slumdog Millionaire

Jordan Schettler

Department of Mathematics
University of Arizona

4/28/11

Outline

Quartic Formula

Nested Expressions

Notebooks and Nonsense

The Riemann Zeta Function $\zeta(s)$

Quartic Formula



- born in India on Dec. 22, 1887; contracted smallpox in 1889

Figure: Srinivasa Ramanujan



Figure: Srinivasa Ramanujan

- ▶ born in India on Dec. 22, 1887; contracted smallpox in 1889
- ▶ became one of the greatest mathematicians of modern times



Figure: Srinivasa Ramanujan

- ▶ born in India on Dec. 22, 1887; contracted smallpox in 1889
- ▶ became one of the greatest mathematicians of modern times
- ▶ extracted deep results from a single out-of-date textbook published in 1856



Figure: Srinivasa Ramanujan

- ▶ born in India on Dec. 22, 1887; contracted smallpox in 1889
- ▶ became one of the greatest mathematicians of modern times
- ▶ extracted deep results from a single out-of-date textbook published in 1856
- ▶ an orthodox Brahmin (Hindu) and a strict vegetarian

- ▶ Ramanujan dreamt of blood drops symbolizing Narasimha's murder of a demon and received visions of formulas.



- ▶ Ramanujan dreamt of blood drops symbolizing Narasimha's murder of a demon and received visions of formulas.
- ▶ He said "An equation for me has no meaning, unless it represents a thought of God,"



- ▶ Ramanujan dreamt of blood drops symbolizing Narasimha's murder of a demon and received visions of formulas.
- ▶ He said "An equation for me has no meaning, unless it represents a thought of God,"
- ▶ but he also remarked that all religions seemed equally true to him.



At the age of 15, Ramanujan was shown how to solve a cubic

$$x^3 + ax^2 + bx + c = 0.$$

At the age of 15, Ramanujan was shown how to solve a cubic

$$x^3 + ax^2 + bx + c = 0.$$

He then solved the quartic equation

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

on his own; one of the roots is given by $x = \dots$

$$\frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}}$$

$$\frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}}$$

$$+ \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}}$$

$$\begin{aligned}
 & -\frac{a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} \\
 & + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}} \\
 & -\frac{1}{2} \sqrt{\frac{a^2}{2} - \frac{4b}{3} - \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}}
 \end{aligned}$$

$$\begin{aligned}
& \frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} \\
& + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}} \\
& - \frac{1}{2} \sqrt{\frac{a^2}{2} - \frac{4b}{3} - \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} \\
& - \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}}
\end{aligned}$$

$$\begin{aligned}
& \frac{-a}{4} - \frac{1}{2} \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} \\
& + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}} \\
& - \frac{1}{2} \sqrt{\frac{a^2}{2} - \frac{4b}{3} - \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} \\
& - \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}} \\
& - \frac{-a^3 + 4ab - 8c}{4 \sqrt{\frac{a^2}{4} - \frac{2b}{3} + \frac{2^{\frac{1}{3}}(b^2 - 3ac + 12d)}{3 \left(2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2} \right)^{\frac{1}{3}}}} + \left(\frac{2b^3 - 9abc + 27c^2 + 27a^2d - 72bd + \sqrt{-4(b^2 - 3ac + 12d)^3 + (2b^3 - 9abc + 27c^2 + 27a^2d - 72bd)^2}}{54} \right)^{\frac{1}{3}}}
\end{aligned}$$

He tried to solve the quintic equation

$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$$

in the following year, but was unsuccessful.

He tried to solve the quintic equation

$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$$

in the following year, but was unsuccessful.

Turns out, there is no quintic formula.

He tried to solve the quintic equation

$$x^5 + ax^4 + bx^3 + cx^2 + dx + e = 0$$

in the following year, but was unsuccessful.

Turns out, there is no quintic formula.

It's not that we just can't find it... we've proven that no such formula exists!

Nested Expressions

In 1904, he was given a scholarship to his hometown college, but he lost it after one year because he neglected all but math.



Figure: Temple in Kumbakonam

In 1905, Ramanujan ran away from home, became ill the next year, and lived in extreme poverty.

In 1905, Ramanujan ran away from home, became ill the next year, and lived in extreme poverty.

He tried to attend the University of Madras in 1906, but failed every subject on his admittance exam except math.

In 1905, Ramanujan ran away from home, became ill the next year, and lived in extreme poverty.

He tried to attend the University of Madras in 1906, but failed every subject on his admittance exam except math.

He became ill again in 1909, was married to a 9 year old girl, and wound up at the *Journal of the Indian Math. Society*:

$$? = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \dots}}}}$$

He posed the above problem, going unsolved for over 6 months.

Note that

$$n^2 - 1 = (n - 1)(n + 1),$$

so

$$n = \sqrt{1 + (n - 1)(n + 1)}$$

Note that

$$n^2 - 1 = (n - 1)(n + 1),$$

so

$$n = \sqrt{1 + (n - 1)(n + 1)}$$

In particular,

$$3 = \sqrt{1 + 2 \cdot 4}$$

Note that

$$n^2 - 1 = (n - 1)(n + 1),$$

so

$$n = \sqrt{1 + (n - 1)(n + 1)}$$

In particular,

$$3 = \sqrt{1 + 2 \cdot 4}$$

$$= \sqrt{1 + 2\sqrt{1 + 3 \cdot 5}}$$

Note that

$$n^2 - 1 = (n - 1)(n + 1),$$

so

$$n = \sqrt{1 + (n - 1)(n + 1)}$$

In particular,

$$3 = \sqrt{1 + 2 \cdot 4}$$

$$= \sqrt{1 + 2\sqrt{1 + 3 \cdot 5}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4 \cdot 6}}}$$

Note that

$$n^2 - 1 = (n - 1)(n + 1),$$

so

$$n = \sqrt{1 + (n - 1)(n + 1)}$$

In particular,

$$3 = \sqrt{1 + 2 \cdot 4}$$

$$= \sqrt{1 + 2\sqrt{1 + 3 \cdot 5}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4 \cdot 6}}}$$

$$= \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \dots}}}}$$

Ramanujan was also a master of continued fractions...

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Ramanujan was also a master of continued fractions...

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Note that

$$\varphi = 1 + \frac{1}{\varphi}$$

Ramanujan was also a master of continued fractions...

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Note that

$$\varphi = 1 + \frac{1}{\varphi}$$

so

$$\varphi^2 - \varphi - 1 = 0.$$

Ramanujan was also a master of continued fractions...

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

Note that

$$\varphi = 1 + \frac{1}{\varphi}$$

so

$$\varphi^2 - \varphi - 1 = 0.$$

Thus

$$\varphi = \frac{1 + \sqrt{5}}{2} = 1.61803398\dots$$

Figure: φ is the Golden Ratio

Ramanujan found a relationship between φ , π , and e :

$$\frac{e^{-2\pi/5}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}$$

Notebooks and Nonsense

“A short uncouth figure... walked in with a frayed notebook under his arm. He was miserably poor... He opened his book and began to explain some of his discoveries... but my knowledge did not permit me to judge whether he talked sense or nonsense.” —Ramachandra Rao



$$\frac{4}{\pi} = \frac{3}{2} - \frac{23}{2^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi\sqrt{3}} = \frac{3}{4} - \frac{31}{3 \cdot 4^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi} = \frac{23}{18} - \frac{283}{18^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi\sqrt{5}} = \frac{41}{72} - \frac{685}{5 \cdot 72^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi} = \frac{1123}{882} - \frac{22583}{882^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{2}{\pi\sqrt{3}} = \frac{1}{3} + \frac{2}{3^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1}{9} + \frac{11}{9^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{3\pi\sqrt{3}} = \frac{5}{49} + \frac{43}{49^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299}{99^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493}{99^6} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

As paper was scarce,
Ramanujan wrote only his final
results on paper.

Figure: A Page in the Notebooks

$$\frac{4}{\pi} = \frac{3}{2} - \frac{23}{2^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi\sqrt{3}} = \frac{3}{4} - \frac{31}{3 \cdot 4^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi} = \frac{23}{18} - \frac{283}{18^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi\sqrt{5}} = \frac{41}{72} - \frac{685}{5 \cdot 72^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{4}{\pi} = \frac{1123}{882} - \frac{22583}{882^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{2}{\pi\sqrt{3}} = \frac{1}{3} + \frac{2}{3^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1}{9} + \frac{11}{9^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{3\pi\sqrt{3}} = \frac{5}{49} + \frac{43}{49^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299}{99^3} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493}{99^6} \cdot \frac{1}{2} \cdot \frac{1.3}{4} + \dots$$

As paper was scarce,
Ramanujan wrote only his final
results on paper.

His notebooks have been the
source of many articles and
books.

Figure: A Page in the Notebooks

$$\frac{\zeta(-2)}{\pi} = \frac{3}{2} - \frac{23}{2^3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{\zeta(-4)}{\pi\sqrt{3}} = \frac{3}{4} - \frac{31}{3 \cdot 4^3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{\zeta(-6)}{\pi} = \frac{23}{18} - \frac{283}{18^2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{\zeta(-8)}{\pi\sqrt{5}} = \frac{41}{72} - \frac{685}{5 \cdot 72^2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{\zeta(-10)}{\pi} = \frac{1123}{882} - \frac{22583}{882^2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{\zeta(-12)}{\pi\sqrt{3}} = \frac{1}{3} + \frac{2}{3^3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1}{9} + \frac{11}{9^3} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{1}{3\pi\sqrt{3}} = \frac{5}{49} + \frac{43}{49^2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299}{99^2} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{992} + \frac{27493}{996} \cdot \frac{1}{2} \cdot \frac{1}{4} + \dots$$

Figure: A Page in the Notebooks

As paper was scarce,
Ramanujan wrote only his final
results on paper.

His notebooks have been the
source of many articles and
books.

There are some results which
took mathematicians many
years after his death to verify.

In February of 1913, Ramanujan wrote to Cambridge mathematician G. H. Hardy and said...

In February of 1913, Ramanujan wrote to Cambridge mathematician G. H. Hardy and said...

“If I tell you

$$1 + 2 + 3 + 4 + 5 + 6 + \cdots =$$

In February of 1913, Ramanujan wrote to Cambridge mathematician G. H. Hardy and said...

“If I tell you

$$1 + 2 + 3 + 4 + 5 + 6 + \cdots = -\frac{1}{12},$$

you will at once point out to me the lunatic asylum as my goal.”

In February of 1913, Ramanujan wrote to Cambridge mathematician G. H. Hardy and said...

“If I tell you

$$1 + 2 + 3 + 4 + 5 + 6 + \cdots = -\frac{1}{12},$$

you will at once point out to me the lunatic asylum as my goal.”

Hardy remarked that Ramanujan's theorems “must be true, because, if they were not true, no one would have the imagination to invent them.”

$$1 + 2x + x^2 \overline{)1}$$

$$\begin{array}{r}
 1 \\
 1 + 2x + x^2 \overline{) 1} \\
 \underline{-(1 + 2x + x^2)} \\
 -2x - x^2
 \end{array}$$

$$\begin{array}{r}
 1 - 2x \\
 \hline
 1 + 2x + x^2 \big) 1 \\
 \quad - (1 + 2x + x^2) \\
 \quad \quad \quad - 2x - x^2 \\
 \quad - (-2x - 4x^2 - 2x^3) \\
 \quad \quad \quad \quad \quad \quad 3x^2 + 2x^3
 \end{array}$$

$$\begin{array}{r}
 1 - 2x + 3x^2 \\
 \hline
 1 + 2x + x^2 \big) 1 \\
 \quad - (1 + 2x + x^2) \\
 \quad \quad \quad \hline
 \quad \quad - 2x - x^2 \\
 \quad - \quad \quad \quad \hline
 \quad \quad \quad (-2x - 4x^2 - 2x^3) \\
 \quad \quad \quad \quad \quad \quad \hline
 \quad \quad \quad \quad \quad 3x^2 + 2x^3 \\
 \quad - \quad \quad \quad \quad \quad \quad \hline
 \quad \quad \quad \quad (3x^2 + 6x^3 + 3x^4) \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \hline
 \quad \quad \quad \quad \quad \quad - 4x^3 - 3x^4
 \end{array}$$

$$\begin{array}{r}
 1 - 2x + 3x^2 - 4x^3 + \dots \\
 \hline
 1 + 2x + x^2 \bigg) 1 \\
 \quad - (1 + 2x + x^2) \\
 \quad \quad \hline
 \quad \quad - 2x - x^2 \\
 \quad - \quad \hline
 \quad \quad (-2x - 4x^2 - 2x^3) \\
 \quad \quad \quad \hline
 \quad \quad \quad 3x^2 + 2x^3 \\
 \quad - \quad \quad \hline
 \quad \quad \quad (3x^2 + 6x^3 + 3x^4) \\
 \quad \quad \quad \quad \hline
 \quad \quad \quad \quad - 4x^3 - 3x^4 \\
 \quad - \quad \quad \quad \quad \hline
 \quad \quad \quad \quad (-4x^3 - 8x^4 - 4x^5) \\
 \quad \quad \quad \quad \quad \hline
 \quad \quad \quad \quad \quad 5x^4 + 4x^5
 \end{array}$$

Thus

$$1 - 2 + 3 - 4 + \dots = \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Thus

$$1 - 2 + 3 - 4 + \dots \text{ “ = ” } \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Therefore

$$- 3(1 + 2 + 3 + 4 + \dots) \text{ “ = ”}$$

Thus

$$1 - 2 + 3 - 4 + \dots \text{ “ = ” } \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Therefore

$$\begin{aligned} & -3(1 + 2 + 3 + 4 + \dots) \text{ “ = ”} \\ (1 + 2 + 3 + 4 + \dots) - 4(1 + 2 + 3 + 4 + \dots) \text{ “ = ”} \end{aligned}$$

Thus

$$1 - 2 + 3 - 4 + \dots = \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Therefore

$$\begin{aligned} & -3(1 + 2 + 3 + 4 + \dots) = \\ (1 + 2 + 3 + 4 + \dots) - 4(1 + 2 + 3 + 4 + \dots) = \\ (1 + 2 + 3 + 4 + \dots) - 2(2 + 4 + 6 + 8 + \dots) = \end{aligned}$$

Thus

$$1 - 2 + 3 - 4 + \dots = \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Therefore

$$\begin{aligned} & -3(1 + 2 + 3 + 4 + \dots) = \\ (1 + 2 + 3 + 4 + \dots) - 4(1 + 2 + 3 + 4 + \dots) = \\ (1 + 2 + 3 + 4 + \dots) - 2(2 + 4 + 6 + 8 + \dots) = \\ & 1 - 2 + 3 - 4 + \dots = \frac{1}{4} \end{aligned}$$

Thus

$$1 - 2 + 3 - 4 + \dots \text{ “ = ” } \frac{1}{1^2 + 2 \cdot 1 + 1} = \frac{1}{4}$$

Therefore

$$\begin{aligned} & -3(1 + 2 + 3 + 4 + \dots) \text{ “ = ” } \\ (1 + 2 + 3 + 4 + \dots) - 4(1 + 2 + 3 + 4 + \dots) \text{ “ = ” } \\ (1 + 2 + 3 + 4 + \dots) - 2(2 + 4 + 6 + 8 + \dots) \text{ “ = ” } \\ & 1 - 2 + 3 - 4 + \dots \text{ “ = ” } \frac{1}{4} \end{aligned}$$

so

$$1 + 2 + 3 + 4 + \dots \text{ “ = ” } (1/4)/(-3) = -\frac{1}{12}$$

Hardy helped arrange for Ramanujan to come to England in 1914 so they could collaborate on research.

Unfortunately, the outbreak of World War I made specialty vegetarian food scarce. This along with stress made Ramanujan fell ill with tuberculosis and a vitamin deficiency.

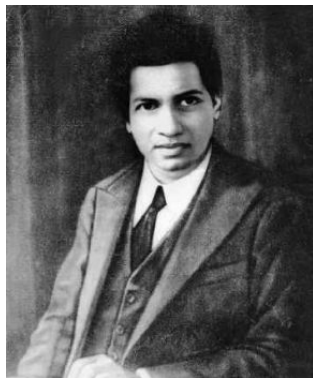


Figure: Photo of Ramanujan

Hardy later wrote “I remember once going to see [Ramanujan] when he was ill... I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one...

Hardy later wrote “I remember once going to see [Ramanujan] when he was ill... I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one...

‘No,’ he replied, ‘it is a very interesting number; it is the smallest number expressible as the sum of two [positive] cubes in two different ways’.”

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

The Riemann Zeta Function $\zeta(s)$

For $s > 1$ define

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

For $s > 1$ define

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

This is THE most important function in mathematics and is **intimately** connected with prime numbers p ; e.g.,

$$\zeta(s) = \left(1 - \frac{1}{2^s}\right)^{-1} \left(1 - \frac{1}{3^s}\right)^{-1} \left(1 - \frac{1}{5^s}\right)^{-1} \cdots = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$$

For $s > 1$ define

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

This is THE most important function in mathematics and is **intimately** connected with prime numbers p ; e.g.,

$$\zeta(s) = \left(1 - \frac{1}{2^s}\right)^{-1} \left(1 - \frac{1}{3^s}\right)^{-1} \left(1 - \frac{1}{5^s}\right)^{-1} \cdots = \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$$

We can extend the definition “smoothly” of $\zeta(s)$ to all complex numbers $s \neq 1$.

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
-----	------------	--------------

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
4	$\pi^4/90$	Stefan-Boltzmann law

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
4	$\pi^4/90$	Stefan-Boltzmann law
3	no simple form known	Apéry's constant

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
4	$\pi^4/90$	Stefan-Boltzmann law
3	no simple form known	Apéry's constant
2	$\pi^2/6$	probability of coprime pairs

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
4	$\pi^4/90$	Stefan-Boltzmann law
3	no simple form known	Apéry's constant
2	$\pi^2/6$	probability of coprime pairs
1	∞	only pole

Table: Special Values of $\zeta(s)$

s	$\zeta(s)$	significance
4	$\pi^4/90$	Stefan-Boltzmann law
3	no simple form known	Apéry's constant
2	$\pi^2/6$	probability of coprime pairs
1	∞	only pole
-1	$-1/12$	bosonic string theory

The Riemann Hypothesis: One of six remaining Millennium Problems whose solution will result in a 1,000,000 dollar prize. The problem is prove that all non-trivial zeros of $\zeta(s)$ lie on the vertical line $x = 1/2$ in the complex $z = (x + iy)$ -plane.

The Riemann Hypothesis: One of six remaining Millennium Problems whose solution will result in a 1,000,000 dollar prize. The problem is prove that all non-trivial zeros of $\zeta(s)$ lie on the vertical line $x = 1/2$ in the complex $z = (x + iy)$ -plane.

Ramanujan may have very well proved this himself had he not died at age 32 in 1920.