Carbon-14 and the San Francisco Skeletons

A **radioactive substance** is a substance that spontaneously decays, meaning that its mass spontaneously converts into other particles. The **half-life** λ of a radioactive substance is the amount of time the substance takes to decay to half of its initial quantity. Carbon-14 is a radioactive isotope commonly used to date fossils, which makes sense because life forms on this planet are primarily carbon based. Its half-life is around $\lambda = 5,715$ years; this long half life makes it convenient for dating.

Use the half-life λ of carbon-14 to write down an exponential decay model $A = Pe^{-rt}$ for the amount A (in grams) of carbon-14 left after t years when starting with P grams. Here P is generic, so you only need to find out what the **rate of decay** r is. (Don't round off r... a small error in the rate could a make a big difference in A.) Hint: if we start with P grams, how much will be left after $\lambda = 5,715$ years? Thinking about this question should give you an equation in which P cancels. You'll then be able to solve this equation for r by using the natural logarithm.

Note: A is a function of t; we call this the **decay function**.

Next, write down a model which uses $1/2$ instead of e as the base. That is, find a model of the form $A = P(1/2)^{kt}$ for some constant k. Is k positive or negative? Is this model more natural than the first? Why or why not? Do the models give the same function? Explain.

Now graph the decay function taking $P = 1$ gram (using either model) and label your axes with appropriate units. What is/are the asymptote(s) of this function and what is/are the corresponding physical interpretation(s)? What domain makes sense for time t? With this domain, what range makes sense; in other words, if we start with $P = 1$ gram, what's the maximum and minimum number of grams we could possibly have over time?

In 1989, some human skeletal remains of several bodies were found at a construction site in San Francisco, California. After testing the bones, it was determined that these skeletons had only 88% of the carbon-14 expected in a living person. Use the above model to determine what year these people died. Hint: first, you need to see how long it would take for some amount P of carbon-14 to reduce to 88% of its original mass. Should the time it takes be shorter or longer than the half-life? Why? Then you'll need to consider what year $t = 0$ corresponds to and what time t corresponds to 1989.

The earliest evidence of habitation in the region which became San Francisco dates to around 3,000 BC. If we found skeletons from this period, what percentage of the expected amount of carbon-14 would we find? The first documented European visit to San Francisco Bay occurred on November 2, 1769. If we found the remains of one of these explorers, what percentage of the expected amount of carbon-14 would we find?

Finally, it didn't seem to matter what P was. Why is this?