

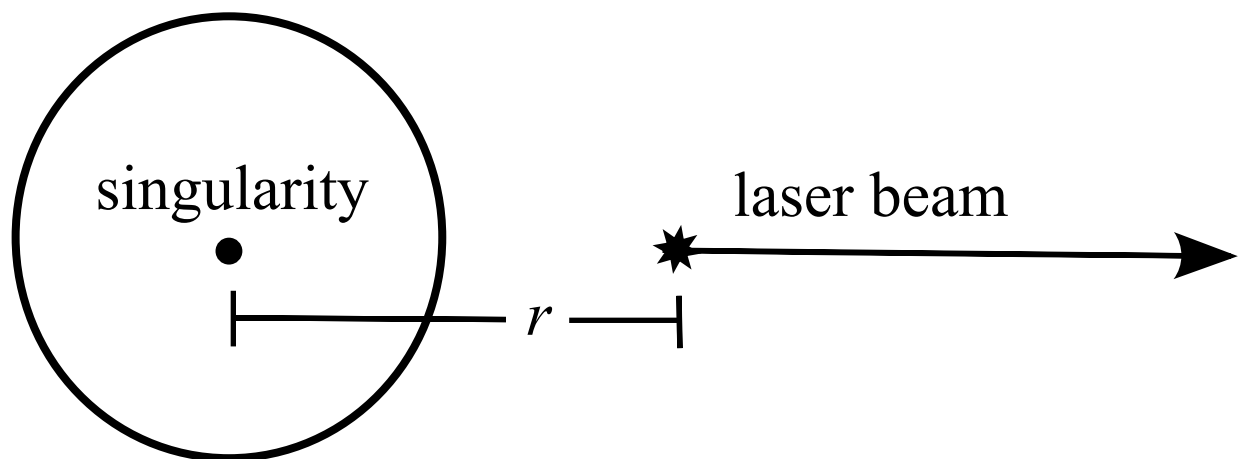
# Cygnus X-1: The Black Hole Lab

The speed (in kilometers per second) of light in empty space is

$$c \approx 300,000 \text{ km/sec.}$$

To put that in perspective, the Earth's circumference is around 40,000 km, so light in space could circle the equator seven and a half times in one second. However, it may surprise you to learn that the speed of light is not constant! Indeed, light slows down slightly when it passes through various mediums like air or glass. In air, the speed of light is very close to (but less than)  $c$ . The speed of light is also affected by intense gravity. Sometimes the force of gravity is so strong that light cannot move forward at all. For example, if a massive (non-rotating) star burns out but cannot support its matter against the force of gravity, then a black hole forms. This means that the star collapses to a single point called the **singularity**. The singularity is a place of infinite gravitational forces and is surrounded by a boundary called the **event horizon**, inside of which light cannot escape and is ever more rapidly pulled into the singularity. Light just outside the event horizon moves very slowly but will eventually escape.

Suppose we shoot a laser beam radially outward at a radius  $r$  in kilometers away from the singularity of a (non-rotating) black hole with a mass around 9 times that of the Sun. See the diagram below.



The closer we are to the singularity, the more the speed of light particles coming from the laser will be affected by gravity. In fact, general relativity tells us that the velocity  $v$  in kilometers per second of the light coming from the laser beam is given by

$$v(r) = c \left( 1 - \frac{26}{r} \right).$$

So  $v$  is a rational function of  $r$ . What is/are the horizontal asymptote(s) of this function (if any)? What's the physical interpretation of the horizontal asymptote(s)? At what radius  $r$  is the event horizon? How do you know? Where is/are the vertical asymptote(s) of this function? What does/do the vertical asymptote(s) represent? Graph this function (include scales and labels on axes) by first finding an appropriate window on your TI-eighty-whatever. Finding a suitable window will not be easy; you must think about values of  $r$  (the domain) and values of  $v$  (the range). The questions you've already answered should help. Is the velocity ever negative? If so, where does this happen and what does this mean? At what radius inside the event horizon is the speed equal to  $c$ ? At what radius inside the event horizon is the speed equal to  $2c$ ? What happens to the speed of light coming from the laser when we're very close to the singularity? What's the speed at the singularity? How can we make sense of negative values of  $r$ ? If we turned the laser around and fired it at the center of the black hole, how long would it take for the beam to go from the event horizon to the singularity? Give an estimate and explain how you arrived at that number.