

The Definite Dozen of Pre-Calculus (First Semester)

1. To add fractions we need a common denominator:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

2. We must be able to foil:

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

and recognize differences of squares

$$x^2 - a^2 = (x + a)(x - a).$$

3. If we have $\sqrt{x} \pm \sqrt{a}$, we multiply by the conjugate $\sqrt{x} \mp \sqrt{a}$ to get $(\sqrt{x} + \sqrt{a})(\sqrt{x} - \sqrt{a}) = x - a$.

4. To find the roots r_1, r_2 of a quadratic $ax^2 + bx + c$, we could...

(a) factor $ax^2 + bx + c = (x - r_1)(x - r_2)$ where $r_1 r_2 = c$ and $-(r_1 + r_2) = b$

(b) use the quadratic formula

$$r_1, r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

5. A point (a, b) is on the graph of a function $y = f(x)$ if and only if $f(a) = b$.
6. The point $(a, 0)$ is an x -intercept when $f(a) = 0$. The point $(0, b)$ is a y -intercept when $f(0) = b$.
7. The average rate of change of a function f over an interval $[x_1, x_2]$ is the slope of the secant line...

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

8. The instantaneous rate of change of a function f at a point x is the slope of the tangent line...

$$\frac{dx}{dy} = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} = f'(x) \quad (\text{the derivative at } x)$$

9. A function $y = f(x)$ is continuous when we can draw its graph without picking up our pencil.
10. The log of a product is the sum of logs: $\log_b(xy) = \log_b(x) + \log_b(y)$.
11. The exponential of a sum is the product of exponentials: $b^{x+y} = b^x b^y$.
12. The log and exponential are inverses: $\log_b(b^x) = x$ and $b^{\log_b(x)} = x$.