Name________________________

Period________________________

Matrix Vector Problem of the Week

Please show all of your work for full and partial credit.

Matrices are frequently used to scale and rotate vectors (applications include physics, statistical analysis, and video games). In this problem we will rewrite vectors in their matrix form, i.e.,

$$\mathbf{u} = \langle x, y \rangle = \begin{bmatrix} x \\ y \end{bmatrix}$$

The rotation matrix equation for two dimensional vectors, $\langle x, y \rangle = \begin{bmatrix} x \\ y \end{bmatrix}$ is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

The matrix in the equation above will rotate a vector about the origin by θ degrees or radians.

1. Draw the vector $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$ on a piece of graph paper. Use the equation above to rotate this vector by 30° ($\frac{\pi}{6}$ radians). Write down the answer in its exact form (this can be done since the trigonometric functions have exact answers at $\theta = 30^\circ$). Draw the rotated vector on the graph paper and annotate (mark) the angle between the two vectors.

   Finally, compute the magnitude of the rotated vector, $\|\mathbf{u}\|$, when $\mathbf{u} = \begin{bmatrix} x' \\ y' \end{bmatrix}$. Is this magnitude the same as the magnitude of the original vector? That is does

$$\left\| \begin{bmatrix} x' \\ y' \end{bmatrix} \right\| = 2\sqrt{13}?$$

2. Prove that the rotation matrix is correct, that is for

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

   prove that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \|\mathbf{u}\| \begin{bmatrix} \cos(\theta + \phi) \\ \sin(\theta + \phi) \end{bmatrix}$$

   when using the standard magnitude angle form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \|\mathbf{u}\| \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}$$

   and $\|\mathbf{u}\| = \sqrt{x^2 + y^2}$. *Hint : Remember the angle sum identities :*

$$\cos(\theta \pm \phi) = \cos(\theta)\cos(\phi) \mp \sin(\theta)\sin(\phi)$$
$$\sin(\theta \pm \phi) = \sin(\theta)\cos(\phi) \pm \cos(\theta)\sin(\phi)$$