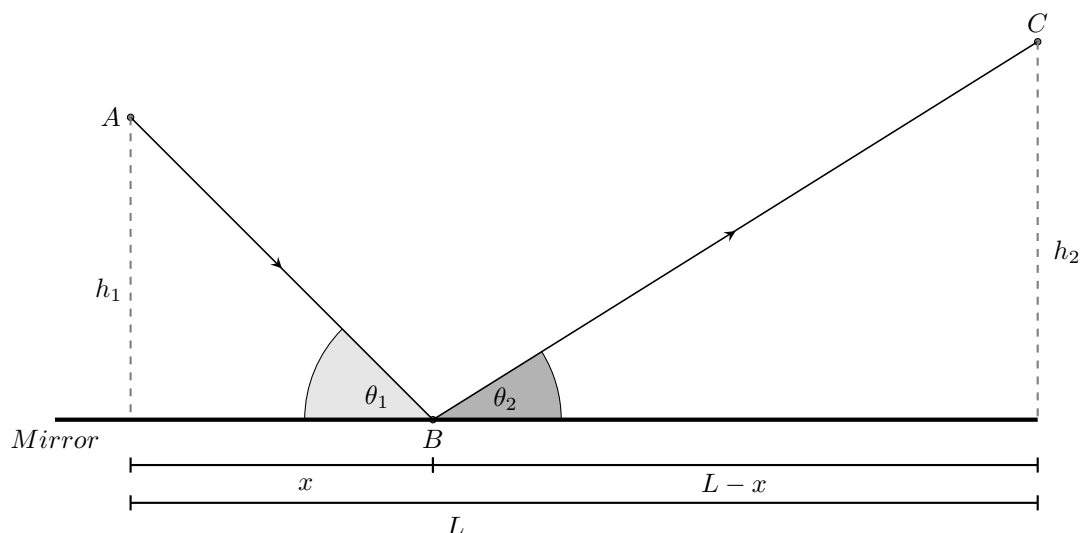




When light reflects from a mirror or glass, the angle of reflection is equal to the angle of incidence. In this exercise we will be walking through the proof of this fact.

Physically, light obeys *Fermat's Principle*, also called *The Principle of Least Distance*. This principle states that light will always travel the least distance possible (or the shortest path). This is subject to some physical realities and constraints. The figure above shows a light ray being directed toward a mirror and then reflected from that mirror. The shortest path between A and C is a straight line between A and C , so one may believe that this should be the path, but that would ignore the fact that we are shining a light at the mirror with some angle (and we have the freedom to change the angle). The problem we are going to solve is *given that we are shining the light at a specific angle, what is the angle of reflection?* This means that given the length AB as being free to choose, and the incident angle θ_1 as being free to choose, what is the shortest total path ABC that exists? Since this is a calculus class, this problem will likely need calculus for the solution.

Mathematically, for the problem above, we have to minimize the equation $AB + BC$.

1. Rewrite the equation $AB + BC$ in terms of x, L, h_1 , and h_2 . This should only require the theorem of Pythagoras for right triangles.
2. Take the derivative of the above equation with respect to x .
3. Rewrite this equation in terms of $\cos \theta_1$ and $\cos \theta_2$. Remember when r is the length of the hypotenuse then $x = r \cos \theta$ for a triangle with one end of the hypotenuse on the origin.
4. Now set the above equation equal to zero (to find minimums and maximums) and show that $\theta_1 = \theta_2$.