

Table 1: Example for $a_1 = 7$

term	is it even?	operation	result
$a_1 = 7$	no	$a_2 = 3a_1 + 1$	22
$a_2 = 22$	yes	$a_3 = a_2/2$	11
$a_3 = 11$	no	$a_4 = 3a_3 + 1$	34
$a_4 = 34$	yes	$a_5 = a_4/2$	17
$a_5 = 17$	no	$a_6 = 3a_5 + 1$	52
$a_6 = 52$	yes	$a_7 = a_6/2$	26
$a_7 = 26$	yes	$a_8 = a_7/2$	13
$a_8 = 13$	no	$a_9 = 3a_8 + 1$	40
$a_9 = 40$	yes	$a_{10} = a_9/2$	20
$a_{10} = 20$	yes	$a_{11} = a_{10}/2$	10
$a_{11} = 10$	yes	$a_{12} = a_{11}/2$	5
$a_{12} = 5$	no	$a_{13} = 3a_{12} + 1$	16
$a_{13} = 16$	yes	$a_{14} = a_{13}/2$	8
$a_{14} = 8$	yes	$a_{15} = a_{14}/2$	4
$a_{15} = 4$	yes	$a_{16} = a_{15}/2$	2
$a_{16} = 2$	yes	$a_{17} = a_{16}/2$	1

Pick a number, any number between 1 and 20 excluding 7 (see the table above for an example). Now if that number is even, divide it by 2. If it is odd multiply it by 3 then add 1. With the new number that you just computed above, determine once again if it is even or odd, then if the new number is even, divide it by 2. If it is odd multiply it by 3 then add 1. Repeat this until you get to 1. You may use your calculator to compute the terms of the series.

Formally, we have the function

$$a_{n+1} = \begin{cases} a_n/2 & , \text{if } a_n \text{ is even} \\ 3a_n + 1 & , \text{if } a_n \text{ is odd} \end{cases} \quad (1)$$

Question: *For any a_1 , a natural number, no matter how large, do you think that eventually the sequence will reach 1, i.e., is there some n such that $a_n = 1$? Why or why not? If not, what could happen instead?*