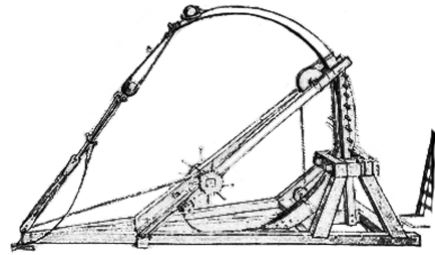


Applications of Quadratics : Catapults

When we model the height of a projectile shot from a catapult as a function of the horizontal distance it has traveled we get a quadratic function. In this activity we use the catapults to shoot a penny into the air. We will measure the time the penny spends in the air and the horizontal distance it travels to determine a quadratic equation for its motion.



For each test run, you will need one person to do each of the following:

- Launch the catapult.
- Watch the penny and mark the spot where it **FIRST** hits the ground. Mark this spot with a piece of tape. Record the horizontal distance the penny went in **centimeters**.
- Time each run. Record how long it took the penny to reach the ground after it was shot by the catapult in **seconds**.

PART I

- Place the catapult on the ground and try a few practice shots. Make sure your catapult does not move. Pull the catapult all the way down each time.
- Once you start to get consistent times and distances, record three of these runs. (*Do not count runs that are not consistent with the others.*) The times and distances for the three runs you chose are:

Time	Distance

- Calculate the average time and distance for these three runs. You will use these average values for the future calculations.

Average Time	Average Distance

4. *Fill in the blanks:* At time $t = 0$ sec, the height of the penny is $h = 0$ cm. At time $t = \underline{\hspace{2cm}}$ the height of the penny is also $h = 0$ cm. In all, the penny spent $\underline{\hspace{2cm}}$ seconds in the air. Since half of this time the penny was rising and half the time the penny was falling, the penny spent $\underline{\hspace{2cm}}$ seconds falling.
5. The vertical distance traveled d of an object in freefall is given by $d = \frac{1}{2}gt^2$ where g is gravity $g = 9.8\text{m/sec}^2 = \underline{\hspace{2cm}}\text{cm/sec}^2$ and t is the falling time. Use this equation to determine the highest point the penny reaches.
- $d = \underline{\hspace{2cm}}$
6. We wish to find a function $h(x)$ representing the height of the penny as a function of its horizontal distance. From the calculation in #5, we know that the y -coordinate of the vertex is $\underline{\hspace{2cm}}$. The x -coordinate of the vertex, will be half of the total horizontal distance traveled. Therefore the x -coordinate of the vertex is $\underline{\hspace{2cm}}$.
7. All together, the vertex of $h(x)$ is $\underline{\hspace{2cm}}$. You also know at least two other points that lie on the graph of $h(x)$. The point $\underline{\hspace{2cm}}$ lies on the graph of $h(x)$.
8. Use your answers from #7 to determine the function $h(x)$ and sketch a graph the function. Recall that the standard form of a quadratic is given by

$$h(x) = a(x - h)^2 + k .$$

$$h(x) = \underline{\hspace{4cm}}$$

PART II

9. The axis of symmetry for the quadratic equation found in #8 is $x =$ _____.
The x -intercepts of the quadratic equation are _____. Label these on the graph sketched in #8.
10. Calculate how far away the catapult should be from the wall in order for the penny to hit the wall at the vertex. Justify your reasoning. (*Hint: Think about your answers in #9.*)
- The catapult should be placed _____ centimeters from the wall.
11. Measure the height of the vertex on the wall and mark with a piece of tape. Test your calculation from #10. How close is the penny to the vertex?

PART III

12. Place the catapult on a raised surface. (You can use your desk, a stack of books, ect.) Measure how high this surface is from the floor. The distance is _____ cm.
13. Give a sketch of the quadratic function describing the motion of the penny when the catapult is place on this raised surface. Label the y -intercept.

14. Using what you learned about translations, write the new equation for your quadratic function $h(x)$.

$$h(x) = \underline{\hspace{4cm}}$$

15. The vertex of $h(x)$ is . Therefore the highest the penny will reach is cm.

16. Calculate where your penny should land when it is shot from this height. Mark this spot on the floor. The penny should land cm away.

17. Shoot the catapult three times and measure the horizontal distances.

Shot Number	Horizontal Distance	Difference between actual distance and distance calculated in #10 (take absolute value)
1		
2		
3		

18. How good was the estimated distance given in #16? If you do not get a good approximation, give some possible (*physical*) explanations for why your calculation did not agree with your results.