G-TEAMS: Graduate Students and Teachers Engaging in Mathematical Sciences

PIs: Joceline Lega, Virginia Horak, and William McCallum, Department of Mathematics, University of Arizona, Tucson, Arizona

G-TEAMS fellows bring their mathematical expertise into K-12 classrooms in a year-long partnership with local mathematics teachers. The goals of the program include: (i) introducing mathematical thinking to K-12 students; (ii) enriching curriculum possibilities for K-12 teachers; (iii) strengthening fellows' ability to communicate mathematical ideas to non-mathematician audiences; and (iv) introducing mathematical modeling techniques to K-12 students in order to enhance their appreciation for the role of mathematics in other disciplines.

In striving to achieve these goals, fellows have to overcome several challenges, such as: (i) discussing abstract mathematical research using limited mathematical content while still remaining faithful to the key ideas; (ii) engaging high school upperclassmen in original problem solving and changing their mindset about learning and exploring mathematics; and (iii) capturing and maintaining the attention of middle school students, particularly in math intervention classes for low achieving students.

The eight G-TEAMS fellows, whose research interests vary from applied mathematics to abstract algebra, are working in partnerships in Tucson and Casa Grande, Arizona at one elementary school, three middle schools, and four high schools.

Chantel Blackburn
Teacher Partner: Janet Liston at Cholla High Magnet School

Challenge: Truthfully representing mathematics to high school students without being too technical

Solution: Connect the mathematics to something tangible and introduce ideas in small pieces

Example: Deriving a discrete Ricker population model with survivorship from assumptions about flour beetle populations

Number of Insects	=	Number of New Insects (New Recruits)	+	Number of Surviving Insects (Survivors)

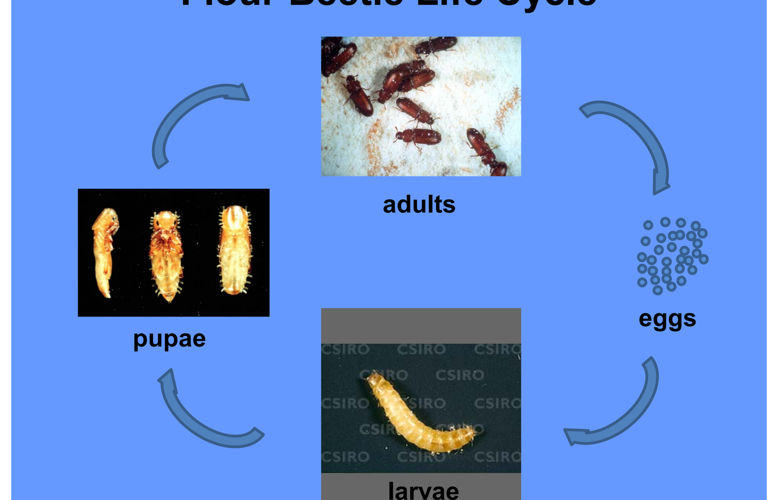
$$\underset{\text{birthrate} \times \text{population size last census}}{x_{t-1}\, b} \times \underset{\text{reduced (negative exponent) by cannibalism}}{e^{-c\,x_{t-1}}} = \text{Number of new insects}$$

$$\underset{\text{probability of death}}{\mu} \rightarrow \underset{\text{probability of survival}}{1-\mu} \rightarrow x_{t-1}(1-\mu) = \text{Number of surviving insects}$$

$$x_t = x_{t-1}\, b e^{c\,x_{t-1}} + x_{t-1}(1-\mu)$$

General Ricker Population Model

Flour Beetle Life Cycle



$$L_t = b\,A_{t-1}\, e^{-c_{el}L_{t-1} - c_{ea}A_{t-1}}$$

$$P_t = L_{t-1}(1-\mu_l)$$

$$A_t = P_{t-1}\, e^{-c_{pa}A_{t-1}} + A_{t-1}(1-\mu_a)$$

Modeling flour beetle populations: 1) given assumptions about life cycles and cannibalism (of eggs and pupae by adults and larvae) and 2) pieces of a general model.

L = Larvae, P = Pupae, A = Adults
b = birth rate, c = cannibalism, μ = death rate
t = time (two weeks)

Outcome: Students think about assumptions and how parameters affect function behavior for exponential growth/decay word problems instead of "not knowing how to start."

Matt Thomas
Teacher Partner: John Willy at Tucson High Magnet School

Challenge: Show students how mathematics is a current and growing field

Solution: Exploring the idea of conjecture

Example: The Collatz conjecture. Define $f(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ n/2 & \text{if } n \text{ is even} \end{cases}$

Example: $n = 3$

$f(3) = 10,\ f(10) = 5,\ f(5) = 16, \ldots$

Sequence generated:

3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, …

Choose an initial value of n and iterate the function. For any number up to roughly 5.764×10^{18}, the sequence generated eventually winds down to a 4 2 1 4 2 1 pattern. It is unknown whether this pattern will eventually be reached by every number. We discussed what would be needed to show the conjecture false, or what would be needed to show the conjecture true. Following this, we discussed Polya's conjecture. It was shown to be false in 1958. The smallest counterexample, however, is 906,150,257.

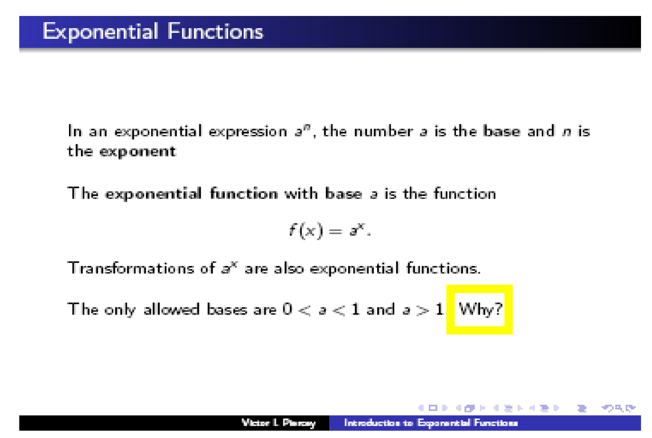
Outcome: This illustrated to many of the students just how troublesome intuition and experimental evidence can be when making general claims. I learned from the process how difficult these kinds of lessons can be.

Victor Piercey
Teacher Partner: Connie Dolezal at Casa Grande Union High School

Challenge: Have disinterested students thinking mathematically

Solution: Brief question during otherwise "ordinary" discussion

Example: The question "why" following definition:

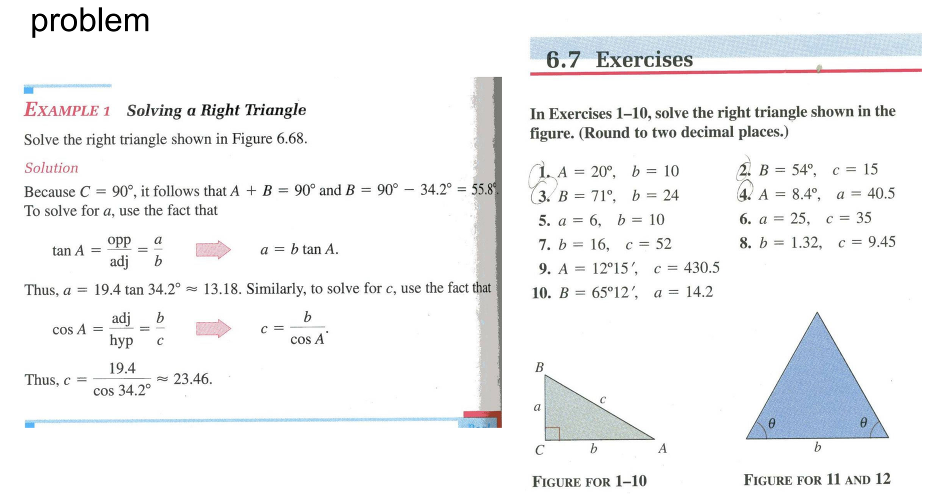


Outcome: A brief, surprisingly lively discussion

Challenge: Help students read math

Solution: Out-loud reading with constant pauses for discussion

Example: Reading an example twice, then solving a similar problem



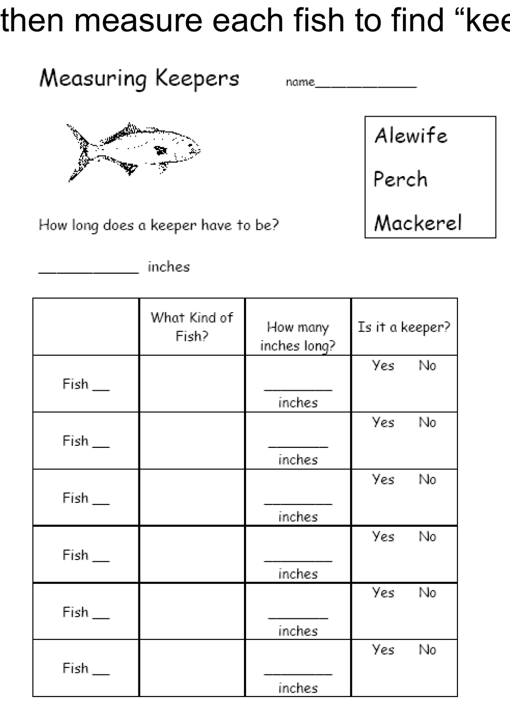
Outcome: Students adapting examples to solve unfamiliar problems

Megan Alexander
Teacher Partners: Christy Erickson and Briana Gryzynger at Gale Elementary School

Challenge: Engaging 25+ first graders for any length of time; addressing greatly varying levels of ability

Solution: Keep talking to a minimum; incorporate a hands-on activity or exciting interaction with every math lesson; have the higher level learners help others when they finish early

Example: Use toy fishing rods with magnets and paperclips on the fish to allow the students to actually "go fishing", then measure each fish to find "keepers".



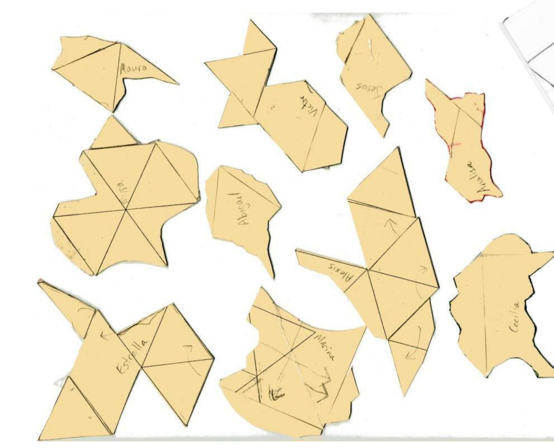
Outcome: They had so much fun with this math lesson that they did not want to stop. When the students have fun and learn at the same time they really begin to love math and build the confidence they need to want to do it again.

Brenae Bailey
Teacher Partner: Arnulfo Velásquez at Wakefield Middle School

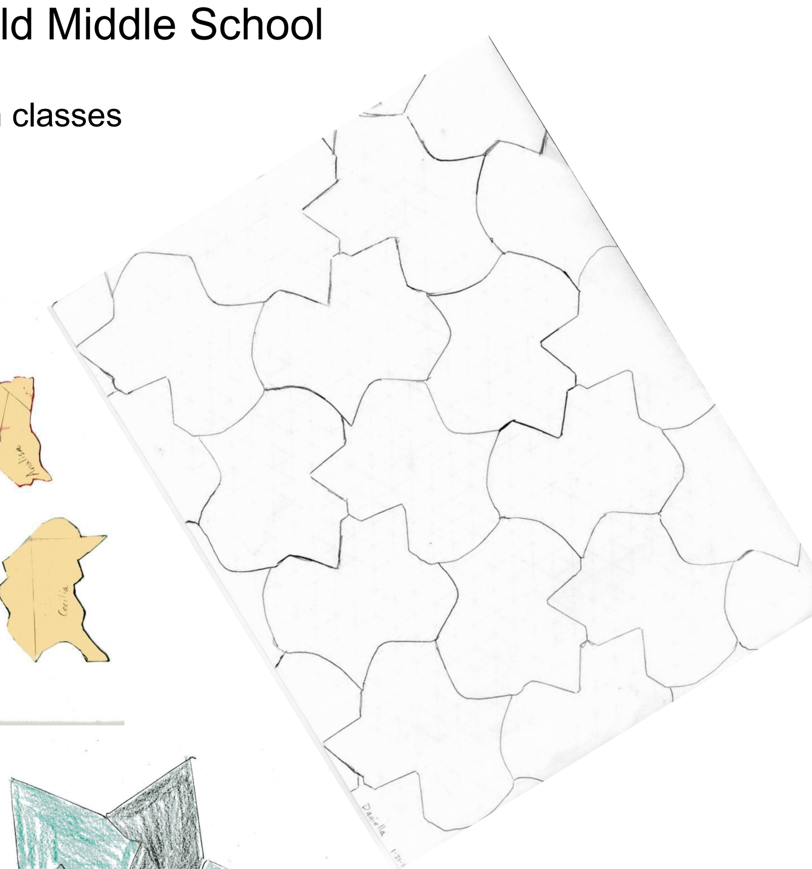
Challenge: Engage low-achieving students in Math Intervention classes

Solution: Do hands-on activities in small groups

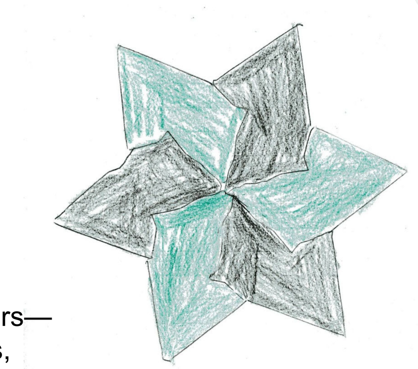
Examples:

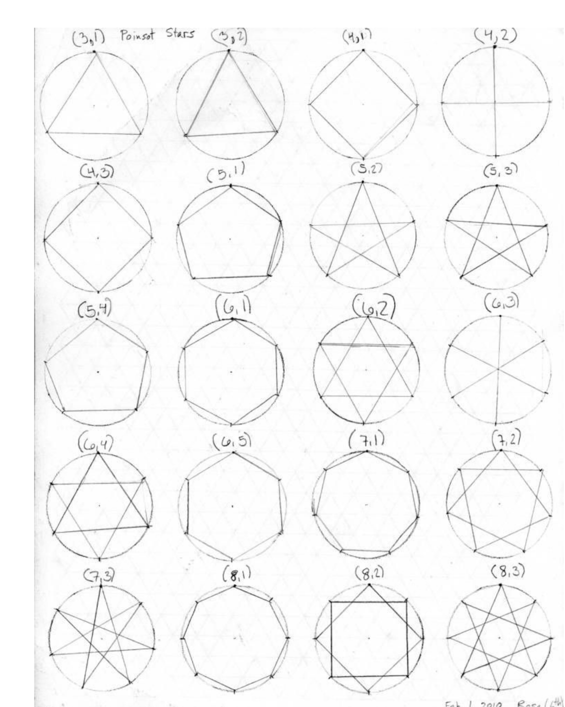


making patterns with sets of tiles—exploring symmetry, comparing areas in terms of simple fractions



tessellations with pattern pieces cut from cardstock—translations and rotations; connections to the art of M. C. Escher





drawing Poinsot stars—following algorithms, recognizing patterns and predicting shapes, classifying objects

Other activities have included 3D modular origami, Möbius strips, and cutting out snowflakes.

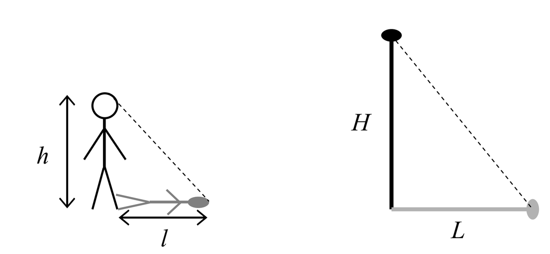
Outcome: Most students are interested and engaged in the activities for at least half of the class period. While I have one group, Mr. Velásquez can work more effectively on the regular lesson with the rest of the students in the class. I hope that these activities will provide some "mental Velcro" to help the students remember the related math concepts when they see them again in more formal lessons.

Grant Peterson
Teacher Partner: Mark Sommers at Mansfeld Middle School

Challenge: Classroom discipline and getting eighth grade students to be engaged

Solution: Design as many hands-on, moving around, doing something activities as we can

Example: Calculate the height of a lamp post or flagpole using similar triangles. Students measure their own height, the length of their shadow and the length of the shadow of the flagpole.



$$\frac{H}{h} = \frac{L}{l} \qquad \text{Solve for } H$$

Outcome: The students were genuinely engaged. They did what we asked of them and seemed to enjoy and care about the problem. It is our hope that in a future lesson, linking our discussion to the memory of this activity will bring the idea of congruent and similar triangles back into focus.