BASIS Tucson North: Algebra II, Precalculus, Calculus BC

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Our Goals

Given that BASIS is an academically advance school, we sought to use the GTEAMS program to

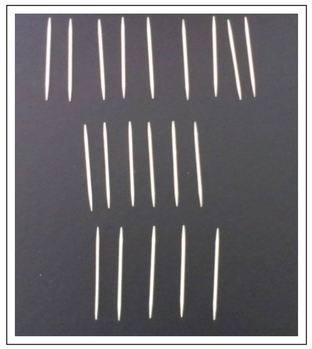
- include more probability and statistics in the curriculum,
- include mathematics topics that relate to outside areas such as the social sciences, and
- to demonstrate how imperfect data is used to understand practical problems.

Games-Nim

In Game Theory, a mathematical algorithm or structure is sought as an explanation of the optimal stategy of game play. A simple game that demonstrates this fact is Nim. In the game of Nim, 2 players are given three different piles of objects (matchsticks). Each player may remove any number of matchsticks from one pile. The winner is the player who picks up the last matchstick.

It turns out that an optimal strategy can be found by looking at the game in terms of binary numbers. Each of the piles has a binary representation for the number of matchsticks. The idea is to leave each column of the binary representations even. This will force the other player to "unbalance" the game, eventually leading to victory for the first player.

Here is a standard game.



Here is the binary representation. Notice not all of the columns are balanced

Number	2^3	2^2	2^1	2^0
9	1	0	0	1
6	0	1	1	0
5	0	1	0	1
Sum	odd	even	odd	even

The optimal strategy is to remove 6 from the first pile to balance the columns

Number	2^3	2^2	2^1	2^0
3	0	0	1	1
6	0	1	1	0
5	0	1	0	1
Sum	even	even	even	even

Peasant Arithmetic

The idea behind this lesson is to demonstrate that there are other ways to multiply two numbers besides our traditional algorithm of lining up the columns and multiplying one decimal place at a time. In this method, it is only necessary to know how to double a number, halve a number, and add numbers.

The process is shown below for $37 \cdot 24 = 888$:

37	24	
74	12	
148	6	
296	3	296
296	2	
592	1	

We now add $592 + 296 = 888$.

With a simple example we can see what is happening in the algorithm. Look at $4 \cdot 4 = 16$. Using dots we see that the multiplication looks like this,

When we double the first number and halve the second, it is just a rearrangement of dots.

One more time we see that we just get all the dots lined up.

To see what happens when an odd number appears we can look at $4 \cdot 3 = 12$.

So we isolate one of the rows of 4 dots and save it for later.

Now we carry out the algorithm on the top portion and add it to the saved (highlighted dots).

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