GTEAMS at Tucson Magnet High School

Israel Vaughn, Brenda Ugalde

Who we are:

Brenda Ugalde

has a BS in Mathematics Education and an MA in Bilingual Education. She has been teaching Secondary Mathematics for 22 years. Brenda believes that all students deserve the opportunity to be successful in school. It is our responsibility, as teachers, to provide an environment that is conducive to learning and accounts for different learning styles.

Israel Vaughn

has a BS in Mathematics and an MS in Mathematics. He is currently a Ph.D. candidate at the University of Arizona College of Optical Sciences. Israel has been involved with several education outreach initiatives over the years. Israel believes that Mathematics education should show students the joy of Mathematics, and impart deeper understanding.

What we teach:

- Honors Pre-calculus II/Calc A
  - Trig and Function review
  - Sequence and Series
  - Matrix and Vectors
  - Conics
  - Limits
  - Derivatives
  - Optimization
- Calculus AB
  - Limits
  - Derivatives
  - Optimization and Related Rates
  - Riemann Sums and Fund. Thm. of Calculus
  - Integrals and Solution Techniques
  - Differential Equations
  - Volumes

Team and Classroom Dynamics

- Brenda typically provides classroom direction and the overall curricular framework. She has very good intuition about the problems with and reactions to new problems from the students.
- Israel typically helps answer questions in new ways, and attempts to give students new problems. He tries to teach some of the deeper concepts. Brenda then helps to formalize ideas and exercises in a way which is appropriate for the students.

Challenges

- New/developing curriculum.
- Motivating students.
- Learning to convey information simply.
- Motivating students.
- Gauging the true ability of students.
- Motivating students.
- Time management.

Support

This material is supported by the National Science Foundation GTEAMS Fellowship under Grant No. 0841234.

The University of Arizona Mathematics department also provided undergraduate students and snacks for the facilitation of the Math Club.

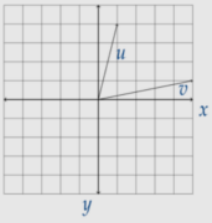
Vectors in Precalc

Use geometric operations on the graph paper below to show how to compute

$$w = 2u - 3v$$

when

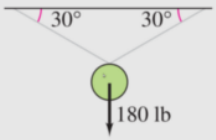
$$u = \langle 1, 4 \rangle$$
$$v = \langle 5, 1 \rangle$$



Find the magnitude and direction angle of

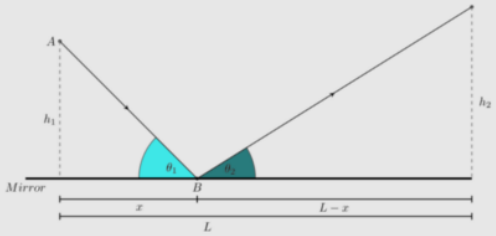
$$\mathbf{v} = \pi(\cos 120°\,\mathbf{i} + \sin 120°\,\mathbf{j})$$

A 180-pound weight is supported by two ropes, as shown in the figure. Find the tension in each rope.

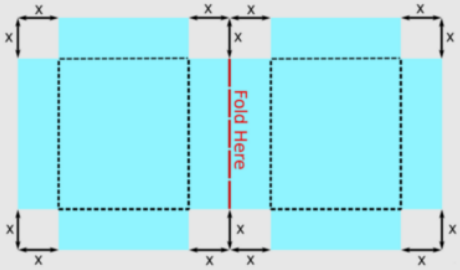


Optimization in Calc AB

When light reflects from a mirror or glass, the angle of reflection is equal to the angle of incidence. In this exercise we will be walking through the proof of this fact.



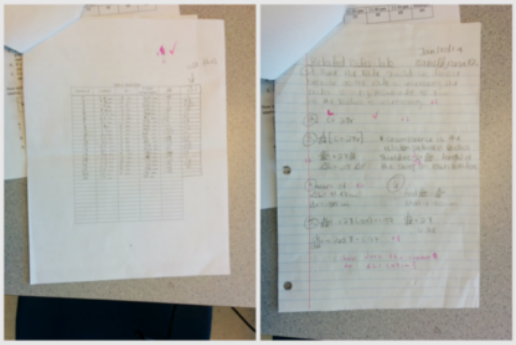
Optimize the volume for the following box folded from the figure shown (the x cutouts are congruent).



Related Rates in Calc AB

$$2\pi r = L$$

$$\frac{d}{dR}\left[2\pi r = L\right]$$



Matrices in Precalc

Prove that the rotation matrix is correct, that is for

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

prove that

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \|u\| \begin{bmatrix} \cos(\theta + \phi) \\ \sin(\theta + \phi) \end{bmatrix}$$

when using the standard magnitude angle form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \|u\| \begin{bmatrix} \cos(\phi) \\ \sin(\phi) \end{bmatrix}$$

and

$$\|u\| = \sqrt{x^2 + y^2}$$

Hint : Remember the angle sum identities.

Sequences and Series in Precalc

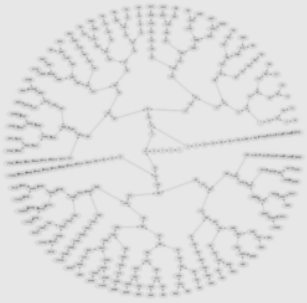
Formally, we have the function

$$a_{n+1} = \begin{cases} a_n/2 & \text{, if } a_n \text{ is even} \\ 3a_n + 1 & \text{, if } a_n \text{ is odd} \end{cases}$$

Question: For any a_1, a natural number, no matter how large, do you think that eventually the sequence will reach 1, i.e., is there some n such that $a_n = 1$? Why or why not? If not, what could happen instead?

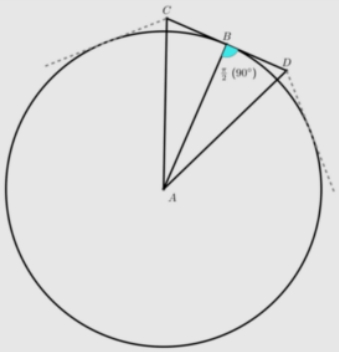
Table: Example for $a_1 = 7$

term	is it even?	operation	result
$a_1 = 7$	no	$a_2 = 3a_1 + 1$	22
$a_2 = 22$	yes	$a_3 = a_2/2$	11
$a_3 = 11$	no	$a_4 = 3a_3 + 1$	34
$a_4 = 34$	yes	$a_5 = a_4/2$	17
$a_5 = 17$	no	$a_6 = 3a_5 + 1$	52
$a_6 = 52$	yes	$a_7 = a_6/2$	26
$a_7 = 26$	yes	$a_8 = a_7/2$	13
$a_8 = 13$	no	$a_9 = 3a_8 + 1$	40
$a_9 = 40$	yes	$a_{10} = a_9/2$	20
$a_{10} = 20$	yes	$a_{11} = a_{10}/2$	10
$a_{11} = 10$	yes	$a_{12} = a_{11}/2$	5
$a_{12} = 5$	no	$a_{13} = 3a_{12} + 1$	16
$a_{13} = 16$	yes	$a_{14} = a_{13}/2$	8
$a_{14} = 8$	yes	$a_{15} = a_{14}/2$	4
$a_{15} = 4$	yes	$a_{16} = a_{15}/2$	2
$a_{16} = 2$	yes	$a_{17} = a_{16}/2$	1



Collatz Graph courtesy of Wikipedia Commons

Math Club



Prove that the area of the n-sided polygon is

$$\frac{pr}{2}.$$