

K-12 Mathematics Teacher Preparation: Local & Global Presentation for GK-12 Teams

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Pathways to AZ certification for teaching mathematics

B.S. or B.A. in Elementary Education	B.S. or B.A. in Mathematics	Teach AZ Master's Degree (M.A.) Education
www.coe.arizona.edu/coe.arizona.edu/current_students/certifications_uu	math.arizona.edu/~smep	www.coe.arizona.edu/ta
Elementary Education (1-8) AEPA MS Math Endorsement AEPA	Secondary Mathematics (7-12) AEPA *Secondary Mathematics Education Program*	Secondary Mathematics (7-12) AEPA
9 Units of mathematics MATH 105 or 112 MATH 302A & 302B 3 units of methods 12 units of ST	34-36 units of mathematics as a Major 25 units of mathematics pedagogy (4 units of methods) 12 units of ST	24 units of pedagogy (3 units of methods) 12 units of ST

Intent of the Common Core

- Fewer topics/standards
- More rigorous content
- The same goals for all students across the nation
- Articulated progressions of topics
- Conceptual understanding and procedural skills stressed equally
- Focus on key ideas, understandings, and skills
- Deep learning of concepts is emphasized - time is spent on a topic and on learning it well. This counters the “mile wide, inch deep” criticism of past state standards
- Skills and concepts are clearly defined, no ambiguities

CCSSM and Teacher Preparation

- Common Core State Standards in Mathematics (CCSSM)...www.corestandards.org
- Newly adopted standards in AZ (2010)
- CCSSM adopted in 46 states
- The changes brought about by CCSSM has impacted (or should impact) the way we prepare teachers at all levels

CCSSM Structure

Elementary K-1-2-3-4-5-6-7-8	High School 9-12
Grade	Conceptual Category
Domain	Domain
Cluster	Cluster
Standard	Standard

Domain → Clusters → Standards

- Domains are overarching big ideas that connect topics across the grades
- Clusters are groups of related standards
- Standards define what students should be able to understand and do; content statements
 - Designed to have progressions of increasing complexity from grade to grade
 - Reflect both mathematical understandings and skills.

K-8 Grade Level Overview



Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Domain

Standard

Cluster

Conceptual Categories – High School

- **The big ideas that connect mathematics across high school**
 - **Number & Quantity**
 - **Algebra**
 - **Functions**
 - **Modeling**
 - **Geometry**
 - **Statistics & Probability**

High School Overview

Domain

Cluster

Seeing Structure in Expressions

A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.*
 - a. Interpret parts of an expression, such as terms, factors, and coefficients.
 - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Standard

Overview of the 2010 Mathematics Standards (Common Core State Standards)

The 2010 Mathematics Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are focused, coherent, and relevant to the real world, describing the knowledge and skills that students need for success in college and careers.

In K-8 (Kindergarten, Elementary, and Middle School) each *grade* contains work on several *domains*, as described in the table below. For example: In Grade 1, the content includes Operations and Algebraic Thinking, Number and Operations in Base Ten, Measurement and Data, and Geometry.

Grade	K	1	2	3	4	5	6	7	8	HS Conceptual Categories
Domains	Counting & Cardinality						Ratios & Proportional Relationships		Functions	Functions
	Operations and Algebraic Thinking						Expression and Equations		Algebra	
	Number and Operations in Base Ten						The Number System		Number & Quantity	
				Fractions						
	Measurement and Data						Statistics and Probability		Statistics & Probability	
	Geometry						Geometry		Geometry	

In High School, the standards are arranged in *conceptual categories*, such as Algebra or Functions. In each conceptual category there are *domains*, such as Creating Equations and Interpreting Functions.

Conceptual Category	Number & Quantity	Algebra	Functions	Geometry	Statistics & Probability
Domains	The Real Number System	Seeing Structure in Expressions	Interpreting Functions	Congruence	Interpreting Categorical & Quantitative Data
	Quantities	Arithmetic with Polynomials & Rational Expressions	Building Functions	Similarity, Right Triangles, & Trigonometry	Making Inferences & Justifying Conclusions
	The Complex Number System	Creating Equations	Linear, Quadratic, & Exponential Models	Expressing Geometric Properties with Equations	Conditional Probability & the Rules of Probability
	Vector & Matrix Quantities	Reasoning with Equations & Inequalities	Trigonometric Functions	Geometric Measurement & Dimension	Using Probability to Make Decisions

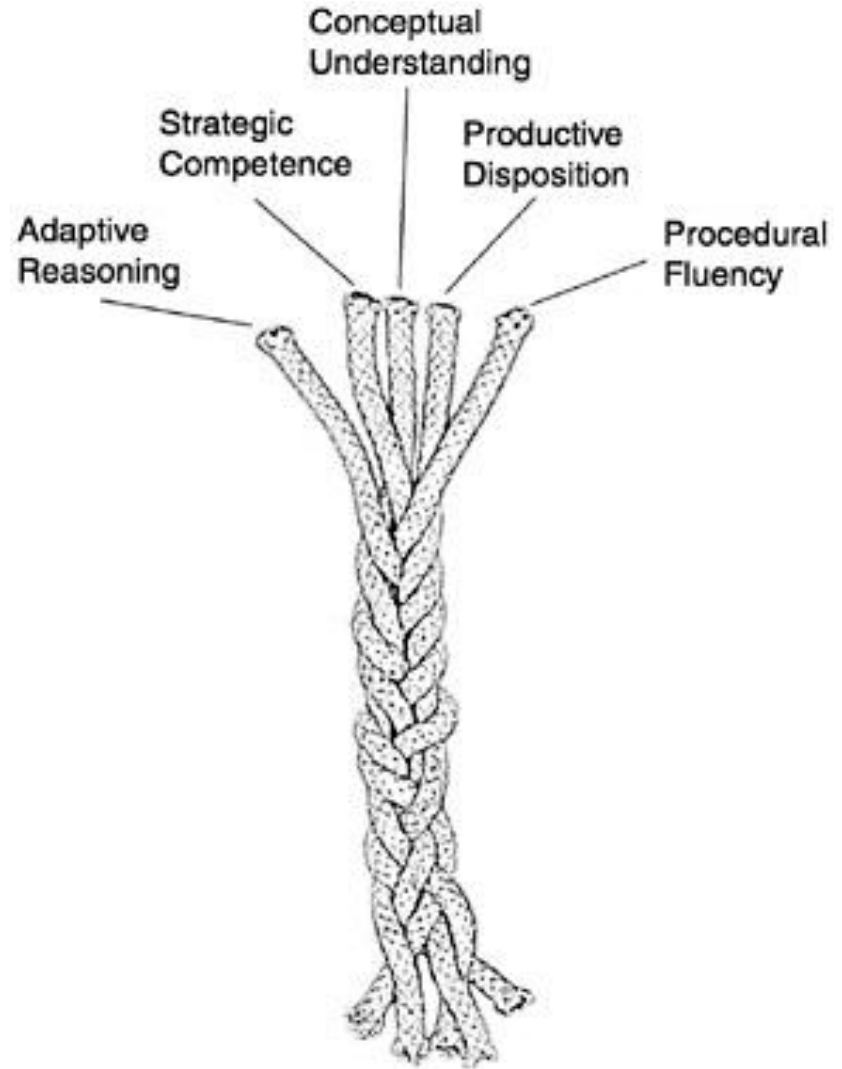
CCSSM Mathematical Practices

The Common Core proposes a set of ***Mathematical Practices*** that all teachers should develop in their students. These practices are similar to ***NCTM's Mathematical Processes***.

Foundations for the CCSS Mathematical Practices

NCTM Process Standards (nctm.org)

- Problem Solving
- Reasoning and Proof
- Communication
- Representation
- Connections



“The Strands of Mathematical Proficiency.” Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press, 2001

CCSS Mathematical Practices

1. **Make sense** of problems and **persevere** in solving them
2. **Reason** abstractly and quantitatively
3. **Construct** viable arguments and **critique** the reasoning of others
4. **Model** with mathematics
5. **Use** appropriate tools strategically
6. **Attend to** precision
7. **Look for** and make use of structure
8. **Look for** and **express** regularity in repeated reasoning



Examples of CCSSM Implementation

Addition

Consider the problem:

$$8 + 4 = \square + 5$$

- **What is the common answer that students give?**
- **34% of 8th graders give “12” as their answer (Boaler & Humphreys, 2005)**
- **How can teachers pose this problem to have a rich discussion about strategies?**

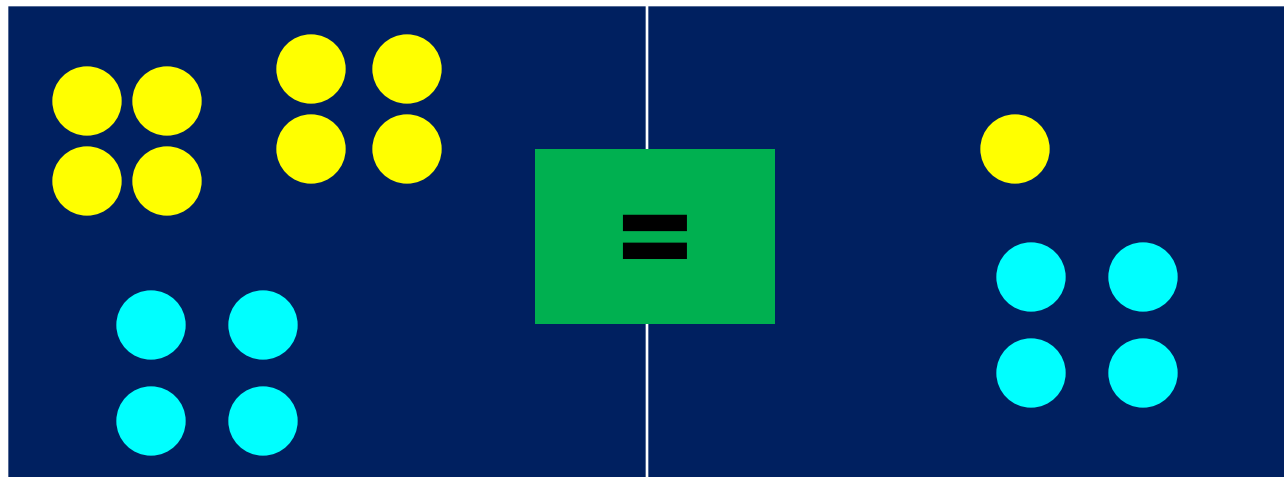
Addition

$$8 + 4 = \square + 5$$

Balanced equation concept

Model $8+4$ on one side and $\square + 5$ on the other side of the equal sign

What questions can teachers ask students about looking at the problem with this model?



Consider $15 + 6 = 14 + 7$

**The question can be posed:
Is this a balanced equation?**

Illustrate the equality by decomposing the numbers.

$$(14+1)+6 = 14+(6+1)$$

What property does this illustrate?

Associative Property $(a+b)+c = a+(b+c)$

CCSSM Operations & Algebraic Thinking

1.OA.3. Apply properties of operations as strategies to add and subtract. *3 Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. (Commutative property of addition.) To add $2 + 6 + 4$, the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$. (Associative property of addition.)*

Subtraction – “Latin-American Algorithm”

- What is the student thinking in the following:

$$\begin{array}{r} 4 \text{ | } 5 \text{ | } 3 \\ - 2 \text{ | } 7 \text{ | } 9 \\ \hline 1 \text{ | } 7 \text{ | } 4 \end{array}$$

Try:

$$\begin{array}{r} 652 \\ - 386 \\ \hline \end{array}$$

- Is the solution correct?
- What understanding is needed for this algorithm?

Subtraction- “Partial Differences Algorithm”

$$\begin{array}{r} 453 \\ - 279 \\ \hline -6 \\ -20 \\ \hline 200 \\ \hline 174 \end{array}$$

Try:

$$\begin{array}{r} 774 \\ - 389 \\ \hline \end{array}$$

What understanding is needed for this algorithm?



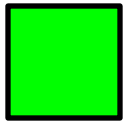
CCSSM Number & Operation in Base Ten

- 3.NBT.2 – Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

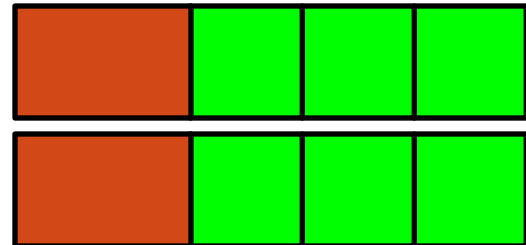
Transitions from Operation to Algebraic Thinking to Algebra

- Model $2(n+3)$
- Use the color tiles to show the distributive property. For example:

 $= n$

 $= 1$

2 groups of $n+3 = 2n + 6$

 $= n+3$

Transition from Operation to Algebraic Thinking to Algebra

Consider $(x+y)^2$ What is the most common response on the AIMS?

$$= (x+y)(x+y)$$

$$= (x*x) + (xy) + (xy) + (y*y) \quad \text{four partial products}$$

$$x + y$$

x	x^2	xy
$+$		
y	xy	y^2

$$(x+y)(x+y) = x^2 + 2xy + y^2$$

Rectangular Model for Multiplication

CCSSM Expressions & Equations

- 6.EE.3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$;*

Teacher Professional Development

- **Mathematics is a high needs area, especially for K-8 levels**
- **The expectations and standards for teaching mathematics at K-12 have evolved and continue to evolve especially with the CCSSM**
- **What ideas do you have for PD?**



Idea:

Mathematical Representation (based on research studies)

**What are the different ways
that we can represent or
model a mathematical idea?**

- **Write down a few ideas**
- **Share your list with the person next to you**
- **Share with whole group**

Representation of Mathematical Ideas



Year, time, cost, place

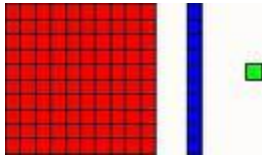
$$2^3 = 8$$

π

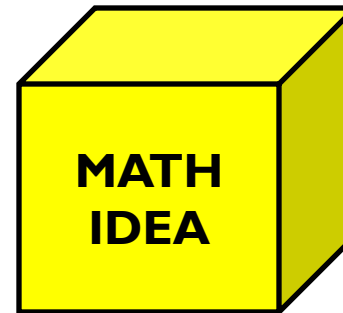
$$(xy)^2 = x^2y^2$$

$$50 - x = 34$$

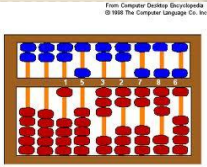
\$ 42.00



Concrete

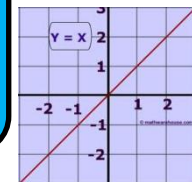
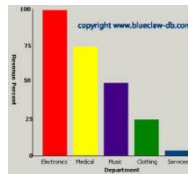


Symbolic



Linguistic

Semi-concrete



Six more than ten, the square of a number is the number multiplied by itself, one ten and six ones, ten percent, greater than

Lesh, Post, & Behr (1987)

Local Efforts in Teacher PD

- **Center for Recruitment & Retention (CRR), Math Dept. (workshops, new teacher induction program)**
- **Math & Science Partnerships (MSP) Grants (NSF)**
- **ITQ Grants (ABOR)**
- **Intel® Math - Institute for Mathematics & Education(IM&E)**

National Council on Teacher Quality (NCTQ)

- Report on the quality and rigor of teacher preparation programs across institutions
- Rating: Model, Good, Weak, Poor
- Washington, D.C. 20005
- www.nctq.org

Recommendations for Teacher Preparation in the Era of Common Core

- **Conference Board of the Mathematical Sciences (CBMS) Publication:**

**The Mathematical Education of Teachers
(2012). Providence RI and Washington DC:
American Mathematical Society and
Mathematical Association of America.**

- **www.cbmsweb.org**

Shortage of Mathematics Teachers

- Shortage of teachers in secondary STEM education
- **Association of Public Land Grant Universities (APLU)** – Science and Mathematics Teacher Imperative (SMTI) – Recruitment efforts to increase students in mathematics and science teacher preparation programs (UA is a new member)

Closure

- **Teaching is highly intellectual work. We need to prepare teachers well.**
- **Teaching mathematics is not business as usual for teachers.**
- **Enacting the CCSS mathematical practices and content well is complex.**
- **Content and practices are part of the tapestry of teaching & learning mathematics.**
- **Teachers need to plan thoughtfully, make sense of the content and mathematical practices, and listen for student thinking.**

Thank you. Questions? Comments?

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