

## Tools for the Common Core Standards

News about tools that are being developed to support implementation of the Common Core State Standards

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## General questions about the Mathematics standards

Posted on [April 2, 2012](#) by [Bill McCallum](#)

If you have general questions about the standards, ask them in a comment on this post. I can't guarantee being able to answer all of them quickly, but by now there is a fairly large community of readers of this blog, some of whom might be able to answer questions, and I'll be sure to keep an eye on it. I'm making this post sticky so it stays at the top of the front page.

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### About Bill McCallum

I was born in Australia and came to the United States to pursue a Ph. D. in mathematics at Harvard University, met my wife, and never went back. I am a professor at the University of Arizona, working in number theory and mathematics education.

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## 171 Responses to *General questions about the Mathematics standards*



[Eric](#) says:

April 2, 2012 at 11:33 am

In my office we have relied a lot on the Progressions Documents to help clarify some confusion with the CCSS. However, many of them are still in draft form, and some are still yet to be released in draft form. When can we expect the Progressions for Geometry, the other part of Measurement and Data, and other Progressions to be released and/or finalized?

[Reply](#)



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[Bill McCallum](#) says:

April 2, 2012 at 4:17 pm

Geometry K-6 and the Measurement part of MD should be out by the end of this month, along with high school Statistics and Probability. All of the progressions will be out in draft form by the end of the Summer.

[Reply](#)

**Beth** says:

April 2, 2012 at 3:52 pm

Can you explain the difference between “mentally add and subtract” and “fluently add and subtract”?

[Reply](#)**Bill McCallum** says:

April 2, 2012 at 4:25 pm

Fluent means “fast and accurate” and mental means “in your head”. A fluent calculation is not necessarily mental; a student could be fluent with a paper and pencil algorithm, for example. And a mental calculation is not necessarily fluent, although I think in all of the instances where this phrase is used in the standards fluency is expected as well. But it might be a valuable exercise for a student to mentally add 2 two-digit numbers, even if the calculus is not very fast.

[Reply](#)**Terri Portice** says:

April 3, 2012 at 7:46 am

What is the scope of teaching money in grades 2 and 4 when the standards occur before decimals are introduced?

2.MD.8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

4.MD.2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

[Reply](#)**Erin Wheeler** says:

April 3, 2012 at 10:26 am

I don't know if this is right or not, but I've been encouraging teachers in 2nd grade to work with just cents or just dollars as an application of whole number addition and subtraction. This could also be a good opportunity to apply the properties of operations and counting on strategies.

In 4th grade I see the connection to decimal fractions in NF.5-7. Would you present these measurement problems using the decimal fractions rather than the algorithms for decimal operations? Would the operations performed on these decimal fractions be limited to what the standards have addressed up to this point (addition and subtraction with like denominators and multiplication of a whole number and a fraction)?

[Reply](#)**Bill McCallum** says:

April 4, 2012 at 9:45 pm

Erin basically has it right. In Grade 2 students are dealing with whole number quantities, so word problems for 2.MD.8 would either deal with whole number amounts of dollars or whole number amounts of cents. This doesn't exclude problems where students have to convert collections of coins and dollars into whole numbers of cents. Indeed, this provides valuable preparation for 4.MD.2.

[Reply](#)



**Erin Wheeler** says:

April 3, 2012 at 8:00 am

2.NBT.7 After reading chapter 1 of Liping Ma's book *Knowing and Teaching Elementary Mathematics*, I think I see a bit better where this standard is going, but I'm not sure if I clearly understand it yet. According to the progressions document on NBT, the standard algorithm is not required in 2nd grade. Mastery of the algorithm in 4th grade grows out the repeated reasoning drawn from the work the students do in K-3.

The inclusion of language such as "composing and decomposing a ten" (hundred, etc.) and the exclusion of language such as "carry a 1" and "borrow" is intentional? Are the standards intending for teachers to avoid explanations that involve the terms carry and borrow? I see how composing and decomposing are more conceptually-sound terms; I just want to make sure I'm interpreting the intention of the standards correctly

[Reply](#)



**Bill McCallum** says:

April 4, 2012 at 9:53 pm

Erin, yes, you have the right interpretation. The standard algorithm builds on a solid understanding of composing and decomposing a ten. Although it is not required in Grade 2, it is not forbidden either. Basically, anything that can get students to a solid understanding of the base ten system in general is worthwhile. For example, carrying and borrowing are words we normally use to describe what elementary school teachers might do with their students. But I think it's fine for teachers to abandon these words in the presence of true conceptual understanding. [Corrected 4/5/2012.]

[Reply](#)



**Erin Wheeler** says:

May 7, 2012 at 2:20 pm

Thanks, Bill, for your response. Also in standard 2.NBT.7, I was wondering what was expected by the statement, "relate the strategy to a written method." What types of written methods does this standard refer to? I was thinking that the students should have a way to capture their mathematical thinking in writing and the problem it represents (knowing how to show a sketch of the base 10 materials and connect it to the written problem  $125 + 241 = 366$ ), but I wasn't sure. Thanks for your help.



**Lisa** says:

April 3, 2012 at 11:35 am

I am looking for some guidance regarding what the expectation is for N-RN.3. It says to "explain why the sum or product of ... is rational; .....that the sum of a rational number and an irrational number is irrational; ..." How much is expected at this

level? If a student is given an item for this standard will the student response include much more than the definition as a way of explaining?

[Reply](#)



**Farshid Hajir** says:

April 5, 2012 at 6:04 am

Here is my interpretation of what this standard requires students to understand and be able to do:

N-RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

First, review of definition: a real number  $x$  is rational if and only if  $bx=a$  for some integer  $a$  and some non-zero integer  $b$ ; in other words, it means  $x$  is the ratio  $a/b$  of two integers, the denominator being non-zero.

(a) why the sum of two rational numbers is rational: suppose  $x,y$  are rational numbers. That means we can write  $x=a/b$  and  $y=c/d$  with integers  $a,b,c,d$  where  $b,d$  are non-zero. Then  $x+y = (ad+bc)/(bd)$ . Since  $ad+bc$  and  $bd$  are integers and  $bd$  is not zero, we've shown that  $x+y$  is rational, which is what we wanted.

(b) why the sum of a rational number and an irrational number is irrational. Suppose  $y$  is rational,  $z$  is irrational and their sum is  $x=y+z$ . Then  $z=x-y=x + (-y)$ . Since  $y$  is rational, so is  $-y$ , so we have expressed the irrational  $z$  as the sum of  $x$  and a rational number. By (a), if  $x$  were rational, then  $z$  would have to be rational too, which it isn't, so  $x$  must be irrational.

(c) why the product of a rational number and an irrational number is irrational. As in (b), suppose  $y$  is a non-zero rational and  $z$  is irrational. Let  $x=yz$  be their product. Since  $y$  is not zero, we can write  $y=a/b$  with  $a$  and  $b$  both non-zero. Then  $z=(b/a)x$  is the product of  $x$  with a rational number. Since the product of two rational numbers is rational (easy proof), the hypothesis that  $x$  is rational would imply that  $z$  is rational, so it must be rejected. Thus,  $x$  must be irrational.

[Reply](#)



**Lisa** says:

April 5, 2012 at 6:29 am

I understand all of what you have indicated. I am not sure how we ask students to verify that they know or understand this. Is this standard to be addressed at a level where these proofs are reasonable expectations for students? The standard seems to be stated at a more introductory level which is what puzzles me regarding assessment.

Thank you for your response.



**Bill McCallum** says:

April 5, 2012 at 6:59 am

It's a good question, Lisa. Because of the work of Deborah Ball and others, we have a good idea of what reasoning and proof can look like in elementary grades: students can explain why the sum of two odd numbers is even, for example, using visual representations of odd and even. In high school, we see geometry as a place where students

learn to produce mathematical proofs. But Middle school has been a bit of wasteland for reasoning and proof. This standard provides an opportunity for that. One way of presenting Farshid's argument to students might be to make the explicit connection with earlier understanding of the relationship between addition and subtraction, so that students can see that  $\text{rational} + \text{irrational} = \text{rational}$  would be the same as  $\text{irrational} = \text{rational} - \text{rational}$ , an impossibility. By the same token,  $\text{rational} \times \text{irrational} = \text{rational}$  would be the same as  $\text{irrational} = \text{rational}/\text{rational}$ , also an impossibility. Then perhaps you could ask "by the way, how do we know that  $\text{rational} + \text{rational} = \text{rational}$ ?" This could be an opportunity to see the formula for fraction addition as not just a computational device, but as a fact about the system of rational numbers (that it is closed under addition).

There's a danger that assessment will drive all this away, of course, attempting to reduce this standard to some mindless exercises; we have to resist that.

[Reply](#)



**Farshid Hajir** says:

April 7, 2012 at 7:06 pm

I agree Lisa's question is a very important one, not just for this standard, but more globally in terms of "how do we engage students in reasoning and develop their ability to justify statements?" Back to this particular standard ... Bill's schematic explanation, cutting to the heart of the matter, is a good example of how a detailed explanation can look "starchy" and obscure the fundamental issue. Regarding getting middle and high school students to reason abstractly, when it comes to this particular standard, in my very limited experience with kids in this age group, what seems to fascinate them most about irrational numbers is that they are characterized by the fact that their decimal expansions don't conclude with a recurring finite pattern. Perhaps the decimal expansion point of view on this standard can be a draw for some students? The question: "if you add a number whose decimal expansion has a repeating pattern to one that does not, what will happen?" is not straightforward and can generate a lot of good discussion among students.



**samantha** says:

April 3, 2012 at 1:55 pm

1. Is the difference between the RI.4 standards (through grade 3) and the Language .6 standards that the reading standard ask students to conclude the definition of a word, while the language standards ask students to apply that knowledge?  
RI.3.4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a grade 3 topic or subject area.

L.3.6. Acquire and use accurately grade-appropriate conversational, general academic, and domain-specific words and phrases, including those that signal spatial and temporal relationships (e.g., After dinner that night we went looking for them).

2. How do RI.4 standards (grades 6-8) and the Language .5 standards differ? Both ask students to determine word meaning by connotation/denotation and of figurative language.

RI.8.4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.

L.8.5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.

Interpret figures of speech (e.g. verbal irony, puns) in context.

Use the relationship between particular words to better understand each of the words.

Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., bullheaded, willful,

firm, persistent, resolute).

[Reply](#)



**Bill McCallum** says:

April 3, 2012 at 2:16 pm

Oops, guess I should have specified that I can only answer questions about the math standards!

[Reply](#)



**Leandra** says:

April 3, 2012 at 5:24 pm

1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Should the unknown be in all positions when dealing with addition of three whole numbers or only in the total?

[Reply](#)



**Bill McCallum** says:

April 5, 2012 at 7:11 am

The previous standard, 1.OA.1, about addition and subtraction word problems, requires the unknown in all positions, whereas this one leaves that open. It seems a natural interpretation to suppose that students can work on problems with the unknown can be in all positions for this standards as well, but that this is not a requirement for assessment purposes.

[Reply](#)



**Brian Cohen** says:

April 5, 2012 at 7:14 am

Bill,

Common vocabulary and definitions have been frequently-reoccurring requests. Many teachers are aware that there are different definitions for some mathematical terms (ex., trapezoid, isosceles, face). Similarly, there are questions about the appropriate language to introduce/use with students (either for developmental reasons, to better foster conceptual understanding, or to assure alignment with the coming tests). For example, should 5th grade students know and use the term “ordered pair,” or “coordinate pair,” both, or something else entirely?

This sort of grade-level specific “Suggested List of Mathematical Language” and “Math Glossary” were provided by our State Dept. of Ed. in the past. However, it doesn’t make sense for 45 states to produce different lists and different definitions for these “common” standards... especially if we will share common assessments. Is there any plan to provide this sort of supporting documents from groups you work with so that they are “common” for all 45 states? If not from your level, it seems very necessary that it be “common” at least within the PARCC states and within the SBAC states. Do they plan to produce such guidance documents?

Thanks,  
Brian

[Reply](#)



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**Bill McCallum** says:

April 13, 2012 at 10:10 am

Brian, this is a great idea, but I don't know of any plan to organize it. Maybe you should start one! 😊

[Reply](#)



**GJordan** says:

April 16, 2012 at 11:38 am

Yes, I agree, Brian should start it, and I'm sure others like me would gladly help.



**Brian Cohen** says:

April 19, 2012 at 8:37 am

Bill,

There seems to be a bit of support out there for the idea. And I don't mind working with my groups on producing first drafts. I have only two questions (and one request):

1. Is there a way to check with PARCC / SBAC that this work isn't already in progress by their folks? (I'd be happy to send an email or make a phone call if you can send me the contact info. for someone who will actually give me an answer!)
2. If I coordinate an effort at drafts, would you (or any of your groups) be willing to look them over? If this sort of vetting is more appropriately done by some other group (ex., PARCC / SBAC or others), can you tell me who?

It would be hugely beneficial for the 45 states who have adopted if there was one "common" public space (ie., website) that was an "official" place to disseminate (

It doesn't seem that there is any good mechanism in place to produce (or disseminate) this sort of inter-state infrastructure. Your blog and the Illustrative Math Project are the closest we have. As a result, 45 states are duplicating efforts and producing different interpretations of the "common" standards. Is there any way to get one "official" website that would function as a \*common\* public space to house \*common\* resources for the 45 states and have \*common\* inter-state discussions? Please?!

I know that a lot this comes down to funding... but it seems like individual states have been given a LOT of RTT money that could go a lot farther (and we could reduce the risk of splintering the common standards) if work could be done once by a group of inter-state folks and then shared on ONE inter-state space!

Thank you for sharing your thoughts, answers, and providing this blog as a common space,  
Brian



**Patrick** says:

April 19, 2012 at 9:16 am

One of the question that I would have is... has anyone taken a poll of the CC states to see which states have adopted the Traditional Model at the HS level vs. the Integrated approach? This could be useful as we begin to share information/resources, etc. Thank you.



**Patrick** says:

April 18, 2012 at 11:15 am

This would be a fantastic resource for our districts/teachers as one of the larger issues that I see with our math instruction in our area is a lack of vertical alignment (both in instruction and in terminology/definitions). Like many of you, our districts will be re-working curriculum maps that align with the Common Core language/content.

[Reply](#)



**Erin Wheeler** says:

April 19, 2012 at 7:48 am

I agree with Brian that this would be very useful. It would be nice for teachers to have common definitions and a common list of key terms to focus on. It would be helpful to see the terms taught in the grades prior to you that are fair game and which terms are introduced after your grade level. I would be willing to help with this if anyone else is interested on working on this. How can we make sure that what is created is connected to the assessment creation process. It wouldn't do much good to give teachers a list of what we think should be taught at each level and then have the test creators working with a different set of vocabulary terms.

[Reply](#)



**sheila shaffer** says:

April 28, 2012 at 5:06 pm

I, too, agree that it doesn't make a whole lot of sense for the CCSS states to be duplicating efforts left and right!! I know that there is a mapping project for the ELA CCSS (There's a fee, I believe!) and the same group has indicated the possibility of beginning a mapping project for math. In the meantime, The Dana Center (funny that it's out of Texas!) has a scope and sequence available for use for CCSS in math. Early elementary teachers, check out Winnipeg School Division Numeracy Project; I think some of the ideas/activities are great for CCSS!!

[Reply](#)



**Jessica McCreary** says:

April 5, 2012 at 8:58 am

Both PARCC and Smarter Balanced indicate a classification system for the content clusters as "major, supporting, and additional." Do you know where that comes from, or who did that, and is it in line with the original intent of the standards?

[Reply](#)



**Bill McCallum** says:

April 13, 2012 at 10:16 am

These are products of the consortia themselves, in their efforts to ensure that the assessments focus on the key

ideas in each grade. I haven't done a thorough or detailed analysis, but I've read through them and by and large I would say that yes, they capture the focus in the standards.

[Reply](#)



**Imhenry9** says:

April 5, 2012 at 8:32 pm

I teach HS Math (specifically Algebra 2). How do you envision how math class would be taught with the Common Core Standards? I think many teachers teach math in a fairly "traditional" way – instructing students on how to do (whatever) and then assign problems to be completed. How is our "mode of business," if you will, going to change?

Thanks – Lisa

[Reply](#)



**Bill McCallum** says:

April 13, 2012 at 10:19 am

Dear Lisa, I don't see the standards as dictating any particular teaching method, but rather setting goals for student understanding. Different people have different ideas about what is the best method for achieving that understanding. That said, I think it's pretty clear that classrooms implementing the standards should have some way of fostering understanding and reasoning, and classrooms where students are just sitting and listening are unlikely to achieve that.

[Reply](#)



**Howard Levine** says:

May 11, 2012 at 11:46 am

I have a question about the statistics component in Algebra 2. How are students going to be expected to determine margins of error by simulation? Also, exactly why is this being done in this course, when more direct, classical methods for confidence intervals and hypothesis tests are discussed in AP Statistics?

[Reply](#)



**Jean** says:

April 6, 2012 at 2:57 pm

Primary teachers become very emotional about the placement of the time and money standards. Kindergarten- no standards on time or money; Grade 1-telling and writing time, no money standards; Grade 2-time and money; Grade 3-no money standards. I have shared my view and would like to share your response which I am certain has more credibility.

[Reply](#)



**Bill McCallum** says:

April 13, 2012 at 10:32 am

Reading time and knowing the value of coins are important life skills, which students could learn in many places:

in the home, in social studies, in science, in mathematics, in history, or in english language arts. There has been a tendency to overload mathematics standards in particular with these life skills, at the expense of more important work on number and operations. Perhaps this was because mathematics standards came along first, so putting these things there was a way of ensuring they were taught. The view of the Common Core is that, used in the right way, they can be tools for learning about number and operations, but they are not mathematics topics in their own right. If kids come to school with knowledge about them, or if there is a way of weaving them into the curriculum that supports the main focus, then that's fine. But too often they become the main focus themselves. The strongest message of the Common Core is: focus on what's important and give it the time it needs, so that kids have a chance to learn it well and progress onto other things. That required paring down previous standards.

[Reply](#)



**Jennie Winters** says:

April 27, 2012 at 12:26 pm

I interpret your comment to mean that the using money to practice counting by 1's, 10's, and 5's in their respective levels or using the clock to practice numeral identification would be reasonable, as those are real-life applications of the mathematics students are expected to be exploring at their level. However, pushing students to count with mixed coins before they have a sense of number composition and decomposition would be discouraged.



**Monique** says:

April 10, 2012 at 11:38 am

Hi Bill,

I am used to there being preliminary work with similar figures in Grade 7 after working with proportions. However, it seems that the intent of the standards is to introduce congruence and similarity with transformations in Grade 8. If this is the case, could you explain the benefit of this approach being taken in the standards?

Thank you for taking the time to answer our questions.

[Reply](#)



**Bill McCallum** says:

April 13, 2012 at 10:37 am

Monique, the Grade 7 standards 7.G.1 gets at some of what you want here: "Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale." It gives an opportunity for student to play around with the informal notion of similarity, without calling it that. There's a mathematical problem with teaching similarity before congruence, in that the notion of similarity depends on the notion of congruency. That is, two figures are similar if you can scale one so that it is congruent to the other. So that's the reason for introducing the formal notion of similarity after congruence.

[Reply](#)



**Lisa** says:



April 11, 2012 at 6:39 am

Again, looking for some guidance — first, I noticed that the progressions for High School Stats is soon to be released. I look forward to that. I am currently studying S-CP.5. It clearly says “in everyday language”. Does this mean that we want students to simply reason about the independence of events.

While I understand what this is saying I am not sure how we assess students in a fair and consistent manner on standards such as this. In everyday language seems to imply that some outside knowledge will be required to discuss these types of situations. Is there a possible sample item that is nearly ready for the illustrative math site that could shed some light on this standard?

[Reply](#)



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**Bill McCallum** says:

April 13, 2012 at 10:42 am

Lisa, we don't have anything in the task bank for S-CP.5 yet, probably because you are right, it's hard to assess. This standard might be best assessed with a modeling task, where students are expected to choose a model for a situation, including making assumptions about whether events are independent or not, and then evaluating their assumptions. We have a few modeling tasks up on Illustrative Math, but not yet for this standard. If you have any ideas for a task, let me know!

[Reply](#)



**Kim** says:

April 13, 2012 at 10:06 am

Hello.

I tried to find the standard associated with simplifying fractions in the lowest terms. However, I couldn't. Can you explain when teachers teach this skill to which grade students?

[Reply](#)



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**Bill McCallum** says:

April 13, 2012 at 10:51 am

Kim, the Standards do not require simplifying fractions into lowest terms, since it is not a mathematically important topic. To quote the [Fractions Progression](#), “It is possible to over-emphasise the importance of reducing fractions .... There is no mathematical reason why fractions must be written in reduced form, although it may be convenient to do so in simple cases.”

Indeed, there are situations where simplifying fractions gets in the way of understanding. For example, insisting that the answer to  $1/10 + 3/10$  be written as  $2/5$  gets in the way of the most important understanding that we want students to come away from this problem with, namely that this addition works the same way as whole number addition, with the unit 1 being replaced by the unit  $1/10$ .

[Reply](#)



**Erin Wheeler** says:



April 16, 2012 at 10:18 am

I have a question about line plots/dot plots. Are they used exclusively for measurement data? I see that they are mentioned as the vehicle for displaying and analyzing measurement data in the progressions document, but I wasn't sure if this meant that you should not use them to represent categorical data. Thanks!

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 10:53 am

Yes, these are for measurement data. The number of dots above a given measurement on the horizontal scale indicates how many times that measurement occurs in the data set. When you do the same sort of thing with categorical data you would call it a bar graph.

[Reply](#)



**GJordan** says:

April 16, 2012 at 12:04 pm

Hi Bill,

Thanks for the opportunity to ask questions for you and the community. I looked for simplifying radicals as an individual learning standard and was unable to find it. Is this purposeful have I overlooked this skill? 8.EE is close and so is N.RN.2 . Is this like the above conversation about simplifying fractions?

[Reply](#)



**GJordan** says:

April 16, 2012 at 12:13 pm

I apologize, Bill has already replied to this topic on another post chain, here's his answer:

“The standard N-RN.2, Rewrite expressions involving radicals and rational exponents using the properties of exponents, could support some work along these lines. But the standards overall try to get away from demanding that students “simplify” things. For example, they don't expect students to find the least common denominator when adding fractions, or to reduce fractions to lowest terms. When thinking about radicals, it's not at all obvious that  $3\sqrt{3}$  is simpler than  $\sqrt{27}$ , and the latter form is more useful for some purposes. For example, you can see that the number is slightly bigger than 5 much more easily from this form.”

Thanks, sorry for the oversight.

[Reply](#)



**Bill McCallum** says:

April 16, 2012 at 12:18 pm

No worries, I had forgotten about that post. Nice to see that I am consistent.



**Bill McCallum** says:

April 16, 2012 at 12:16 pm

You've got the right standards there, particularly N-RN.2. I would also include A-SSE.3. Students should be able to rewrite  $\sqrt{12}$  as  $2\sqrt{3}$  and vice versa, but neither of these is simpler than the other. The emphasis in the standards is on transforming expressions into different forms for a particular purpose, as described in A-SSE.3. So yes, it's similar to the conversation about fractions. The word "simplify" does not occur in the standards (except in one grade level introduction, which was an editing error).

[Reply](#)



**Lane** says:

May 2, 2012 at 9:18 pm

That's a great idea about "simplify" not being simpler! One of the biggest hurdles for Algebra 2 students, though, is working with rational expressions/equations when they are inexperienced with finding that least common denominator. They want to multiply all the denominators together and end up with 4th and 5th power polynomials in the numerator instead of easily factorable quadratics. How do the successful countries handle a problem like that? Do they skip the least common denominator or is it something we are overlooking?

[Reply](#)



**Danielle Goedel** says:

April 17, 2012 at 6:43 am

Regarding 8.G.3: When using coordinates to show the effects of transformations should students be able to extend that knowledge to rotation and dilation points other than the origin? It seems that distance from the origin is the basis so I was wondering if they should be able to extend that knowledge.

[Reply](#)



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**Bill McCallum** says:

April 26, 2012 at 10:55 am

Well, you can have a rotation about any point and a dilation from any point, so I guess those would be included. Note that in 8.G.8 students are using the Pythagorean theorem to calculate the distance between any two points.

[Reply](#)



**Rob Lengacher (@amishrob)** says:

April 17, 2012 at 8:31 am

Hi Bill,

The progressions document on Ratios and Proportional Reasoning 6-7 has been incredibly helpful as I attempt to dig into 6.RP.3a-d. However, I have two questions about 6.RP.3d:

1) I'm not sure I understand the expectation for students where it states, "manipulate and transform units". The standard is clear up to that point, but this phrase seems to suggest actions other than converting. Am I trying to read too much into it?

Or is this suggesting some application of the standards for mathematical practice that I am not seeing?

2) From the last paragraph on p. 7 of the progressions document, it seems that converting units between measurement systems (customary to metric and vice versa) using ratio reasoning is an expectation for sixth graders. Is this correct?

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 11:44 am

1) It could mean also understanding that when you multiply a quantity expressed in seconds by a quantity expressed in meters per second, then you get a quantity expressed in meters. Also dealing with things like minutes times meters/second, or feet times acres (connecting with 6.G.2).

2) Well, the progression suggests it as a possibility for 6.RP.3d, not a requirement. It's a natural thing to do, but the standard does not give an explicit list of which unit conversions are expected, so there is room for curriculum writers to use their judgment here.

[Reply](#)



**Cobb** says:

April 17, 2012 at 9:42 am

Thanks so much for taking time to repond to all of these questions. What it meant when in third grade NF 3 special cases? 3NF3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

[Reply](#)



**Tad W** says:

April 17, 2012 at 12:30 pm

Not sure what "special cases" may be, but the denominators in Grade 3 are limited to 2, 3, 4, 6, and 8. Although the CCSS does not limit the fractions to be discussed in Grade 3 to proper fractions, there are still infinitely many fractions with those denominators. However, I would suspect that the limit of equivalent fractions in Grade 3 will be basically the proper fraction situations, that is,  $1/2 = 2/4 = 3/6 = 4/8$ ,  $1/3 = 2/6$ ,  $2/3 = 4/6$ ,  $1/4 = 2/8$  and  $3/4 = 6/8$ .

[Reply](#)



**Bill McCallum** says:

April 17, 2012 at 12:54 pm

Tad has it right. The idea is to limit to situations where you can see the equivalence by direct reasoning from the definition of a fraction, but not get into the general way of seeing equivalence. For example, you might see that  $1/2$  is equivalent to  $3/6$  using a tape diagram divided into 2 and then into 6. But you wouldn't get into  $3/6 = 3 \times 1/3 \times 2 = 1/2$ .

[Reply](#)



**Heather** says:



April 17, 2012 at 2:30 pm

Bill,

We are eagerly awaiting the release of the Geometry Learning Progressions and the measurement part of the Measurement and Data Learning Progressions. As we're analyzing 3.MD.2 and 4.MD.2, we're wondering about instruction related to "masses of objects" with elementary school students. Since elementary students typically focus on objects on Earth, the distinction between mass and weight is not emphasized in most elementary classrooms, and many elementary school teachers often use the terms mass and weight interchangeably. However, we don't see any references to weight in the CCSS (other than K.MD.1-2), and we don't want teachers to inadvertently foster misconceptions. Should we follow the more technical route and ONLY refer to "mass" when measuring or estimating with g and kg? Or, is it okay to meet students where they are and allow teachers to use "weight" (or to interchangeably use mass/weight) since weight is more commonly used in authentic contexts. Your insight would be greatly appreciated. Thank you in advance.

[Reply](#)



---

**[Bill McCallum](#)** says:

April 26, 2012 at 11:50 am

This seems to me a case where the judgement of the teacher or curriculum writer is important. On the one hand, there are opportunities to model correct usage here (e.g., talking about the mass of the earth). On the other hand, you don't want to forbid people from using common language such as "I weigh 60 kg".

[Reply](#)



**Brian Cohen** says:

April 26, 2012 at 12:53 pm

As teachers and curriculum writers exercise this judgment, I hope they turn to Mathematical Practice 6 (Attend to Precision) for some guidance. It seems to me that there is nothing incorrect about measuring mass in pounds (though it is definitely not conventional). "Weight" is measured on a scale and is affected by gravity. "Mass" is measured on a balance, which means that changing gravitational pulls would not affect it. For example, my weight on earth would be different than my weight on the moon; however, my mass would be the same on earth as on the moon.

With this, it seems that using mass and weight interchangeably would not be 'Attending to Precision,' as they are not synonymous. I would not have a problem using pounds as a unit for measuring mass, as long as we are measuring on a balance.



**Liz Yockey** says:

April 18, 2012 at 11:56 am

Thanks for doing this, Bill! I see a big emphasis on equations, tables, and graphs that represent proportional relationships in the standards, and I am wondering where the first place is that you might introduce an algebraic relationship between variables that is not proportional (e.g.  $y = mx + b$  where  $b$  is non-zero). Is that something to be included in 6.EE.9 (Use variables to represent two quantities in a real-world problem that change in relationship to one another) or is it something that first appears in 8th grade Functions or somewhere in between?

[Reply](#)



**Liz Yockey** says:

April 18, 2012 at 11:58 am

Clarification: I see where solving equations of the form falls, but I am wondering about relationships between variables that change, that you might graph on a coordinate plane.

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 11:52 am

I think this is certainly something that could be included 6.EE.9.

[Reply](#)



**Mark** says:

April 18, 2012 at 4:51 pm

In your post on Arranging the High School Standards into Courses, you lay out a 159 day pacing guide for Algebra I. In California, the state testing window opens about 145 days into the school year. Plus, there are many interruptions that take instructional days away. What would you recommend if we were trying to build a 130 day blueprint?

[Reply](#)



**Patrick** says:

April 18, 2012 at 6:13 pm

Mark and Bill,

This is very similar to that of New York State with the number of days that teachers have to teach. The 130 day layout would be beneficial (although I would assume that with only 130 days, it would make sense to “borrow” the days from all of the areas). What also interests me is building this kind of a pacing guide with SLO’s and Interim assessments scattered throughout. We are pushing for at least three Interims throughout the year. We would also need to build in some time for the scoring of the interims and SLO’s (especially SLO’s given that teachers who have a stake in student achievement should not grade their own students’ papers... which leads to regional scoring).

Anyone else thinking about this as well?

[Reply](#)



**Ellen** says:

April 26, 2012 at 5:59 pm

Mark -

You mention state testing (CSTs). In high school, the Common Core State Standards will not be tested with CST End of Course Exams. The summative assessment in high school (CA is part of SMARTER Balanced Assessment Consortium) will be at the end of junior year, and all standards except those marked with a (+) can be tested. Our current testing model will be different, with schools and districts having some flexibility over the testing window in the 11th grade.



**sheila shaffer** says:



April 28, 2012 at 5:19 pm

Also from NY.... Sounds like you're talking 4-8 testing and not Regents. SLO's will be used for those subjects who don't have CCSS. NY has joined PARCC (but not adopted the assessments, yet, hence the pilot questions on this year's state tests as a back-up in case PARCC on-line isn't ready for next year!) They are also looking at the major assessment being given at about 90% completion of the year's instruction and are backing off of quarterly assessment being ready for delivery in 2012-2013.



**Kaycie says:**

April 19, 2012 at 8:55 am

I work with high school mathematics teachers and would like to ask a question though I'm sure others will follow as we continue to decompose what is within the standards. I do not see any direct mention of the angles of polygons within the CCSS. Of course, there is G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. We can easily see how angles of polygons could be included in this standard as well as more generalized ideas. Would you clarify this for us?

[Reply](#)



**Brian Cohen says:**

April 20, 2012 at 5:11 am

Kaycie,

Maybe what you are looking for is included in 8.G.5 (Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.)?

Brian

[Reply](#)



**Kaycie says:**

April 20, 2012 at 12:43 pm

I agree that this standard from eighth grade will be the beginning place for the conversation regarding interior and exterior angles of polygons by concentrating on triangles, but it does not seem to give rise to other polygons with  $n$  sides where  $n$  is greater than or equal to 3.



**Monique says:**

April 24, 2012 at 10:04 am

I was wondering about this also. My interpretation is that it is appropriate to informally explore the angle sum rule for an  $n$ -sided polygon in 8th grade using triangles, potentially having students make a conjecture about a formula. However, formal teaching of it does not seem required nor forbidden in 8th grade. Perhaps a future high school geometry progression document will "shed light on the problem."



**[Bill McCallum](#) says:**



April 26, 2012 at 11:56 am

Monique is correct: there are opportunities in the standards to explore the angle sum formula for polygons, but it is not explicitly required.

[Reply](#)



**Kaycie** says:

April 19, 2012 at 1:33 pm

What does it mean to “prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.” (A.REI.5) Does that mean a formalized proof of some type – algebraic or by argument or flow chart or what? Our teachers know how to present solving systems of equations by linear combination easily enough and how to have students demonstrate their understanding of that skill, but they are stumped by what is expected of students with a proof that linear combinations provides the same solution to the original system of equations.

Any insights you can provide will be greatly appreciated.

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 12:05 pm

The proof does not have to be a standalone formal object, it could well be embedded in a presentation of the method. There's a difference between showing how to use a method and explaining why it works. I think it's a question of the language used while presenting the method. For example, you might say that if  $(a,b)$  is a solutions tp a system, then the left side of each equation is equal to the right side of each equation, so adding the left sides gives the same number as adding the right sides. That's a bit different from just showing the mechanics of adding the equations.

[Reply](#)



**Robert Springer** says:

April 21, 2012 at 4:03 pm

Bill,

My wife and I are teaching elementary school teachers the principles of Common Core Math in the lower grades. As part of these courses, we give teachers a syllabus with links to relevant material on the internet. You have provided invaluable illustrations of every standard and I would love to provide a link for the teachers to supplement many of the standards being taught (for example: “3.NF Locating Fractions Greater than One on the Number Line”. However I have not found a simple way of linking to specific discussions and illustrations. We give the teachers the general URL (<http://illustrativemathematics.org/standards/k8>) to the content standards and tell them the detailed path for finding a particular discussion or illustration. My bet is that most are not willing to follow these fairly tedious paths.

Is there any way to link to this material without going to home page and using the cascading menus?

A related question is whether there would be any problem if I copied the text and illustrations of interest and put it in the

syllabus.

Robert Springer

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 12:06 pm

We will have the ability to link to tasks soon. As for the copying question, it's fine as long as you acknowledge the source, and follow the rules in the license (bottom left of the page).

[Reply](#)



**Ashli** says:

May 7, 2012 at 4:45 pm

Hi Robert,

I just wanted to follow up and let you know that the ability to directly link to a task at [illustrativemathematics.org](http://illustrativemathematics.org) is now available. We are also working on providing a pdf link for all tasks, but that is a work in progress for older tasks.

Cheers,

Ashli

[Reply](#)



**Leandra** says:

April 22, 2012 at 1:51 pm

Dr. McCallum,

I need help with clarifying the fluency with addition and subtraction facts in K-2.

K- Fluent with addition & subtraction w/in 5

1- Fluent with addition & subtraction w/in 10

2- Fluent with addition & subtraction w/in 20 AND knows from memory single-digit to 9+9 (add only)

We are working on standards based report cards for our 1-2 grade levels. We have standards based in Kinder already. This year in kinder, we listed the standard as shown and then, for assessment purposes only, we used flash cards to assess fluency. (We were sure to use number talks, images, and manipulatives to ensure understanding).

When we started working on standards based for 1 and 2 we ran into some confusion, because we had at first thought in K and 1 we should be "flash card fluent" under 5 and under 10 respectively, but then in 2nd it says know from memory for the ones they should be "flash card fluent" with and the word fluent has a slightly different meaning.

What wording could be used on a report card to differentiate these skills for parents? And if the K and 1 should not be "know from memory" how should teachers assess the facts in kinder and first?

[Reply](#)



**Tad Watanabe** says:



April 24, 2012 at 4:42 am

In November 2000 issue of Teaching Children Mathematics, Susan Jo Russell discussed what NCTM meant by “fluency.” She writes (p. 154):

—

Fluency, as used in Principles and Standards, includes three ideas: efficiency, accuracy, and flexibility.

- Efficiency implies that the student does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of subproblems and making use of inter- mediate results to solve the problem.
- Accuracy depends on several aspects of the problem-solving process, among them, careful recording, the knowledge of basic number com- binations and other important number relation- ships, and concern for double-checking results.
- Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible to be able to choose an appropriate strategy for the prob- lem at hand and also to use one method to solve a problem and another method to double-check the results.

—

I don’t know if this view is consistent with the CCSS writers’ view, but I like (yes, my personal preference) this view on fluency. It should also be noted that “fluency” seen from this perspective does not necessarily mean “quick.”

I also think it is interesting that the CCSS distinguish “fluency within 20” and “knows from memory ” up to  $9+9$ . This seems to suggest that students should be fluent with calculations like  $13+5$  and  $18-3$  using their understanding of the meaning of operations and number sense. But, for  $1+1 \dots 9+9$ , the CCSS seems to expect students to “just know” the facts. I also like the fact that the CCSS puts “memorization” AFTER fluency. I think if students become fluent (as explained by Russell), they will remember basic facts, too.

[Reply](#)



**Leandra says:**

April 26, 2012 at 8:00 am

Thanks Tad. So in your opinion, would you start assessing “know from memory” addition facts only in kinder under 5 and under 10 in 1st as a progression toward the 2nd grade standard?

Or would you just expect them to be able to do them “unhaltingly” but not necessarily from memory?

For our second graders we are using flash cards to assess during individual student interviews and we expect them to know the fact within 3 seconds. Thoughts?



**Bill McCallum says:**

April 26, 2012 at 1:52 pm

First, “fluently” refers to how you do a calculation, whereas “know from memory” means being able to produce the answer when prompted without having to do a calculation. In CCSS, “fluent” means “fast and accurate.” The sort of flexibility that Tad is talking about is coded into many of the standards that are not explicitly about fluency, so it is part of the standards as a whole. I note that Tad says “fluent” does not necessarily imply “quick”, whereas I have said that it does imply “fast”. So there seems to be a bit of disagreement there, although maybe not that much; “fast” for a Kindergartner is not as fast as “fast” for a 2nd grader. If a Kindergartner adds numbers within 5 by saying the starting number and then counting on at a normal verbal pace, without hesitation, and gets it right each time, then I would say the student is fluently adding within 5.

[Reply](#)



**Tad Watanabe** says:

April 26, 2012 at 2:17 pm

I think it is a matter of how quick is quick enough. For example, in Grade 1, if a child thinks, without hesitation, “9 + 4 is 9 and 1 is 10 and 3 more is 13” it will be quick enough to be fluent. However, it is definitely not as quick as simply recalling the fact  $9+4=13$ .

On the other hand, Russell’s definition of fluency may be a bit problematic. For example, if a 2nd grader is adding  $9 + 8$  by counting on 8 times from 9, without hesitation, is he fluent? I would say no because I would want 2nd graders to be moving away from inefficient counting strategy to obtain the correct answer.

As for assessing Kindergarteners, my inclination is not to worry about “know from memory” since the CCSS does not say it explicitly. I may still use flash cards to pose questions, but I would be assessing not how quickly students give me the correct answers but how they seem to be obtaining the answers.



**Leandra** says:

April 23, 2012 at 2:31 pm

Okay, I would start this post with “one more question” but I can’t guarantee it will be my last. 😊

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Is this referring to both 2-D and 3-D shapes, and which shapes should we expect them to model as opposed to draw and vice versa?

[Reply](#)



**Tad W** says:

April 25, 2012 at 4:51 am

I think both “building” and “drawing” are types of “modeling.” So, it’s not a matter of whether students should model or draw.

Modeling (drawing and building) with different materials allow Kindergarten students to start paying attention to the parts that make up shapes. So, students should engage in modeling activities using tiles (pre-made shapes), sticks (focusing on sides of polygons), drawing by connecting dots on dot grid (seeing the vertices of polygons), etc.. Kindergarteners don’t have to specify those components by formal terms, but those experiences help them move from just seeing the whole shape to being able to (eventually) analyze the components of shapes.

CCSS seems to put a lot more emphasis on 3-D shapes, so I imagine students should have some experiences with building 3-D shapes (with blocks, empty boxes, sticks and clay balls, etc.). But, drawing, i.e., representing 3-D shapes on a 2-D medium is probably too much for Kindergarteners.

[Reply](#)



**Paige** says:



April 24, 2012 at 12:42 pm

1.G.2 Compose two-dimensional shapes or three dimensional shapes to create a composite shape, and compse new shapes from the composite shape.

The second part of this standard confuses me. What exactly does that entail?

I believe I understand the first portion as combining two right triangles to make a square, which would be your “composite” shape. So would you take the new square (the composite shape) and another square to make a rectangle? (Obviously this is just one example.)

p.s. Love this blog. Really looking forward to the progressions in Geometry!

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 1:15 pm

Yes, your example certainly fits the standard. Also, situations where you use those interlocking cubes to build up lines, then put the lines together to make rectangles. One of the purposes of this is to get students accustomed to holding more than two different levels of structure in their minds.

[Reply](#)



**Jeannine** says:

April 26, 2012 at 7:42 am

Can someone clarify F.LE.1a? “Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals”. Do the students need to be able to do a formal proof of this? Or will they be given an example and they have to justify it?

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 1:11 pm

I think it’s somewhere in between formal proof and giving an example. I can imagine a lesson that starts with examples, and then asks students to “look for and express regularity in repeated reasoning” (MP8) and come up with a general algebraic argument. That argument could look something like this: if  $f(x) = mx + b$ , then  $f(x)$  grows by  $mh$  over any interval of length  $h$ , because  $f(x+h) - f(x) = m(x+h) + b - mx - b = mh$ . Initially students might just look at cases where the length of the interval is 1, and where  $m$  and  $b$  are given numbers, and then build up from there.

[Reply](#)



**Lane** says:

May 4, 2012 at 7:14 pm

This looks like a precursor for the definition of a derivative...from a typical 13- or 14-year old Algebra I student. I’m biting my lip on this one. Our 17- 18- year- old AP Calc students find that definition challenging

to wrap their minds around when it is given to them. Is there something I'm misinterpreting here as far as expectations for Algebra I?



**Angela says:**

April 26, 2012 at 7:52 am

Could someone provide clarification on A.REI.7: "Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically." What is the definition of a quadratic used here? Does it include circles, ellipses, hyperbolas? Or just parabolas?

[Reply](#)



**Bill McCallum says:**

April 26, 2012 at 1:06 pm

Examples involving circles, ellipses, and hyperbolas certainly fit with the standard. However, the standard does not require that you cover all of these cases.

[Reply](#)



**Terez says:**

April 26, 2012 at 8:03 am

Our district is looking for clarification on: A.CED.2: "Create equations in two or more variables to represent relationships between quantities; Graph equations on coordinate axes with labels and scales." When this strand refers to "two or more variables" does that infer that we need to solve equations with three variables? Would this include teaching matrices?

[Reply](#)



**Bill McCallum says:**

April 26, 2012 at 1:02 pm

Note that this standard is not about solving equations, but about writing them. It does not include matrices. For example, it includes writing the equation  $Q = 100 + 5t$  to represent a quantity that grows at a constant rate, and then graphing  $Q$  against  $t$ . The "two or more variables" allows for situations where you might have a quantity that depends on more than one variable. For example, the balance in a bank account might depend on both the number of years  $t$  and on the interest rate  $r$ ,  $B = 1000(1+r)^t$ .

[Reply](#)



**Patricia says:**

April 26, 2012 at 10:21 am

Our department has been trying to interpret the strands and we are having a difficult time. One question we have is regarding N.Q.3. (Choose a level of accuracy appropriate to limitations on measurement when reporting quantities). Does this mean that students should be able to calculate relative error and percent error?

[Reply](#)



**Bill McCallum** says:

April 26, 2012 at 12:21 pm

No, that exceeds the standard. The standard just mean students should be able to choose the right level of accuracy. For example, if the legs of a right triangle are measured using a ruler marked in 10ths of a centimeter, and you calculate the hypotenuse using the Pythagorean theorem, it does not make sense to report the answer to two decimal places.

[Reply](#)



**David Thiel** says:

April 30, 2012 at 8:59 am

Would teaching rules for computing with significant figures also exceed the standard? Science teachers would be pleased if such rules were taught in first-year algebra.



**Janice** says:

April 27, 2012 at 7:05 am

I'm trying to understand the placement of this standard N-RN.3 in Unit 5 (Appendix A) and what the teaching of it entails in the context of the Unit. It appears nowhere else in any course. I'm thinking it may be pointing to understanding and working with radicals since radicals consistently appear while working with quadratics. If it is pointing to radicals, why was it not introduced in Unit 4 when students are working with quadratic expressions and equations?

[Reply](#)



**Bill McCallum** says:

May 9, 2012 at 4:30 pm

I agree it sticks out where it is. I see Appendix A as a first attempt at arranging the high school standards into courses, and there are no doubt many opportunities for people to come up with better ideas. This standard is really a capstone for work on the number system from earlier grades.

[Reply](#)



**Scott Koch** says:

April 27, 2012 at 10:25 am

has anyone developed potential timelines for teaching with the common core standards? Our district is finding it difficult to get through all the topics in time for state tests which occur in April.

[Reply](#)



**Ashli** says:

May 7, 2012 at 5:29 pm

Hi Scott,

This post might be what you are looking for: <http://commoncoretools.me/2012/03/16/arranging-the-high-school->

[standards-into-courses/](#)

Cheers,

Ashli

[Reply](#)



**Jennie Winters** says:

May 1, 2012 at 9:30 am

Can you clarify if “A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.”

includes the expectation of using a linear equation to solve an exponential equation in Algebra I/Math I?

In Appendix A it states: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as  $5x = 125$  or  $2x = 1/16$ .

[Reply](#)



**Bill McCallum** says:

May 10, 2012 at 9:41 am

A-REI.3 itself is silent on the arrangement of topics into courses, so it can't by itself be interpreted as saying which types of equations occur in Algebra I/Math I and which occur later. Appendix A is a sample arrangement of the standards into courses, but it does not carry the force of the standards themselves; that is to say, a curriculum can follow the standards without following Appendix A. I'm not sure exactly what you mean by “using a linear equation to solve an exponential equation”. The example you quote from Appendix A is about understanding the laws of exponents, not about linear equations.

[Reply](#)



**Bridgett** says:

May 1, 2012 at 9:40 am

Hello!

I have a question about mode in the CC. I see median and mean mentioned in grade 6, but no mention of mode. Where should mode come into play, if at all?

Thanks so much for all of your insights. Excellent blog!

[Reply](#)



**David Thiel** says:

May 3, 2012 at 10:57 am

Bridgett,

The usefulness of the mode depends on the nature of the data. If the data are discrete with a limited number of values (e.g. the number of pets each of our students own), then the mode may tell us something interesting. If the data are more continuous with many different valued (e.g. the heights of our students measured in cm), then the mode may be

of little use.

6.SP.5d speaks of relating choice of measure of center to shape and context. That is the heart of it. Students should understand conceptually when the mode is and is not useful as a summary measure.

[Reply](#)



**Trish Despagni** says:

May 1, 2012 at 11:13 am

Bill,

First, thank you for answering all of our questions posted here. My questions are regarding Geometry in Grade 5.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.  
5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

- How detailed should the hierarchy of 2-dimensional shapes be? Should kite be included?

- Also, what is the definition of a trapezoid? Is it “only one” pair of parallel sides or “at least” one pair of parallel sides? This would affect the hierarchy diagram.

Note: According to Van de Walle (2010) in *Elementary and Middle School Mathematics*, 7th Ed., “Some definitions of trapezoids specify only one pair of parallel sides, in which case the parallelogram would not be a trapezoid. The University of Chicago School Mathematics Project (UCSMP) uses the “at least one pair” definition, meaning that parallelograms and rectangles are trapezoids” (p. 411).

Thanks,

Trish

[Reply](#)



**Tad Watanabe** says:

May 1, 2012 at 2:26 pm

I am very curious to know the answer to the trapezoid definition question. However, in the final report of the National Math Panel, you find this statement:

“By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).” (p. 20)

So, it appears that the NMP is saying that the definition of trapezoids is “at least” one pair of parallel sides.

[Reply](#)



**Joe Ratasky** says:

May 10, 2012 at 3:29 pm

The trapezoid issue has come up in our county before. The way I deal with the possibility of having two distinct definitions is by making the students and teachers aware that some people agree on “only one pair” whereas some people agree on “at least one pair” of parallel sides. I leave it open to discussion. Often times people say they like math because there is always a “correct answer.” But this is not always true. This is one

case where discussion and understanding of other opinions is essential.

Our state (Florida) defined trapezoids in our testing glossary. So though we had great discussions, we let the students know that ultimately on our state test, we will define trapezoids as having only one pair of parallel lines. I imagine more clarity will come with test item specification from either PARCC or SBAC.



**Bill McCallum** says:

May 16, 2012 at 6:52 am

Thanks for your patience while I catch up on answers to these questions. This discussion as already laid out the issues pretty well. The fact is that there are two competing definitions out there, and no authority (including CCSS) says which one to use. That said, I think there are good mathematical reasons for choosing inclusive definitions (e.g., a rectangle is a trapezoid, a square is a rectangle). It's hard to imagine a situation where you want to state a property of a trapezoid that depends on one pair of sides not being parallel, so in practice what you say about trapezoids will be true of parallelograms as well. It's awkward to have to keep saying "and this is also true of parallelograms" every time you make a statement.

[Reply](#)



**Lane** says:

May 16, 2012 at 7:14 am

The inclusive language also inspires connection between formulas. Since the trapazoid's area is the average of the bases times the height, then we could say the same for a parallelogram. If students can "see" so many of the formulas as simple variations of  $bh$  (or  $Bh$  for volume), there is less to memorize.



**Leandra** says:

May 2, 2012 at 11:44 am

3.OA.7 Fluently multiply and divide within 100 and know from memory all basic facts up to  $9 \times 9$ .

In addition to knowing from memory the basic multiplication facts are 3rd graders becoming fluent with 2-digit by 1-digit multiplication so long as the product is 100 or less? For example they should be expected to find  $27 \times 3$  or  $15 \times 4$ , but not  $47 \times 5$  or  $85 \times 2$ .

[Reply](#)



**Joe Ratasky** says:

May 10, 2012 at 3:19 pm

Leandra,

I don't think that is the intention of the standard. I believe they just used the "within 100" example to cover all of the possible products of one digit factors. In fact, looking in 3.OA.7 it does say products of two one digit numbers.

[Reply](#)



**Leandra** says:

May 10, 2012 at 7:04 pm

Joe, so in your opinion there is no double digit multiplication in 3rd grade except by multiples o ten? Or am I

maybe missing it in another standard?



**Bill McCallum** says:

May 11, 2012 at 6:27 am

Multiplication of 1-digit by 2-digit numbers is included as long as the product is less than 100. From the glossary (on page 85 of the standards):

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example:  $72 \div 8 = 9$ .

[Reply](#)



**Joe Ratasky** says:

May 11, 2012 at 9:54 am

3.OA.7

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

So Bill, in looking at the glossary, the standard and your post, it sounds as if this is an issue where the standard is not limiting, but stating the minimum we should expose our students to and hold them accountable for. We could go beyond. For instance, they should leave 3rd grade with a working knowledge of all products from two one-digit numbers, but they may also be exposed to working with products of a one-digit and two-digit number within 100. The glossary doesn't seem to define for us what IS included, nor does it tell us what is NOT included. This is another issue where the standard is very open to interpretation. Also in looking at 4th grade standard 4.NBT.5, that is a standard that does specifically mention multiplication with a one-digit by a multi-digit number, which the 3rd grade standard does not do.



**Joe Ratasky** says:

May 12, 2012 at 9:53 am

I'm making this reply after I have already posted my reply below:

One thing that does seem contradictory to the statement of multiplying withing products of 100 is 3.NBT.3, multiply one-digit numbers by multiples of 10 in the range 10-90. Even one example listed in the progression document (K-5 Number and Operations in Base Ten) shows 3 groups of 50, which would of course result in a product greater than 100.

My interpretation then would be that for students to explore equal groups of two-digit numbers is expected in third grade. Which makes sense when students are learning conceptually of what the meaning of multiplication is, as well as using place value throughout operations. (Modeling 3 groups of 25 would not be that different conceptually than modeling 6 groups of 4.) But for fluency, third graders would be expected to learn related multiplication and division facts of one-digit by one-digit factors.



**Liz Yockey** says:



May 2, 2012 at 12:36 pm

I am wondering about the difference between 6.NS.8 (Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.) and 6.G.3 (Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.)

I see that 6.NS.8 could be interpreted much more broadly than 6.G.3, so I guess I am wondering what are appropriate 6.NS.8-type tasks that are NOT already 6.G.3-type tasks? I would not include graphing proportional relationships or relationships between an independent and a dependent variable as there are other standards about those situations.

I checked [illustratedmathematics.org](http://illustratedmathematics.org) and didn't find any illustrations for these. Looking forward to having some soon! All of this has been so helpful!

[Reply](#)



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**Bill McCallum** says:

May 11, 2012 at 6:22 am

Liz, thanks for pointing this out, I'll let the Illustrative Mathematics team know!

[Reply](#)



**Lane** says:

May 2, 2012 at 9:02 pm

I'm wondering why I don't see as much "push" for CCSS math as there is for English-Literacy when so many more students require remedial math than reading when entering college. Is this just in Missouri?

[Reply](#)



**Erin Wheeler** says:

May 4, 2012 at 8:12 pm

No, Lane, it seems to be that way in NY too. So I don't think it's just Missouri. Many of our RTTT state sponsored training sessions have been on ELA, and we have had minimal training on the CCSS math to this point. I don't know why that is.

[Reply](#)



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**Bill McCallum** says:

May 11, 2012 at 6:15 am

Lane, I don't know why either, but IM&E is planning to organize more workshops like the upcoming one in New Orleans.

[Reply](#)



**Erin Wheeler** says:



May 15, 2012 at 9:27 am

I'm looking forward to the workshop in New Orleans. We really need the content specific math information and plans to move professional development forward. I appreciate the work that IM&E is doing to address these needs. Thank you!



**Pam** says:

May 7, 2012 at 6:04 pm

My question is about the following standard: MCC.7.SP.3 ... measuring the difference between the centers by expressing it as a multiple of a measure of variability. The Illustrative Math Project task for this standard states, "The difference in means of 1.5 million is only on the order of  $1/3$  of the MADs, indicating that the means are not far apart in light of the variation in populations among the states." Since comparing the difference in means to a variation of the MADs is not something that I've ever seen done before (let alone in 7th grade), where would be a good place to go to get a good understanding of why it makes sense to compare these statistics in this manner. In essence, I need to make sense of this myself, before I can make it make sense to my 7th graders. HELP!

[Reply](#)



**Cathy Kessel** says:

May 8, 2012 at 6:00 pm

Pam, have you already looked at the [6–8 Statistics and Probability Progression](#)? Please let me know if that helps.

[Reply](#)



**Pam** says:

May 9, 2012 at 5:57 am

Yes, I have. It lays a good foundation for the first part of that standard, but says nothing about measuring the difference between means or medians as a multiple of mean absolute deviation. It's the second part of the standard that is giving me trouble.



**CK** says:

May 10, 2012 at 4:05 pm

Pam, I don't like to play statistics educator when my background and experience is in math, so I'll refer you to the following two things from the American Statistical Association.

Description of ASA education resources: <http://www.amstat.org/education/pdfs/EducationResources.pdf>.

The Meeting Within a Meeting (MWM) Statistics Workshop for Mathematics and Science Teachers will help middle and high school teachers teach the increased statistics content in the Common Core State Standards. The MWM statistics workshop will be held in conjunction with the Joint Statistical Meetings on Tuesday, July 31 and Wednesday, August 1 at the Hilton San Diego Bayfront with separate middle and high school strands. The registration fee is \$50, which includes materials and refreshments. Optional graduate credit and limited lodging reimbursement is also available. More information and registration for the MWM workshop is available at <http://www.amstat.org/education/mwm/>.



**Cathy Kessel** says:

May 10, 2012 at 4:23 pm

Pam, sorry to reply to myself but I don't see a spot to reply to you.

I don't like to play statistics educator when my background and experience is in math, so I'll refer you to the following two things from the American Statistical Association.

Description of ASA education resources: <http://www.amstat.org/education/pdfs/EducationResources.pdf>.

The Meeting Within a Meeting (MWM) Statistics Workshop for Mathematics and Science Teachers will help middle and high school teachers teach the increased statistics content in the Common Core State Standards. The MWM statistics workshop will be held in conjunction with the Joint Statistical Meetings on Tuesday, July 31 and Wednesday, August 1 at the Hilton San Diego Bayfront with separate middle and high school strands. The registration fee is \$50, which includes materials and refreshments. Optional graduate credit and limited lodging reimbursement is also available. More information and registration for the MWM workshop is available at <http://www.amstat.org/education/mwm/>.



**Jo Walker** says:

May 8, 2012 at 8:30 pm

Should line plots be plotted on a line or a line segment. The example in the progressions is a line segment, but all of the examples I have seen in textbooks are plotted on a line. Which is correct? Or does it depend on the context of the data?

[Reply](#)



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**Bill McCallum** says:

May 11, 2012 at 6:14 am

I'm not sure I understand the question. Is this about how the endpoints of the line are indicated? That's a matter of the convention chosen within a particular curriculum, I think (also a matter of context, as you say).

[Reply](#)



**June** says:

May 9, 2012 at 6:14 am

There is a sample grade one "core aligned" module which asks students to use an inch and cm ruler. Doesn't the core introduce ruler measures in grade 2?

[Reply](#)



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**Bill McCallum** says:

May 11, 2012 at 6:07 am

You are correct that ruler measure is not required in Grade 1. In grade 1 students start to work with the idea of using a standard length unit to measure, as in 1.MD.2:

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

[Reply](#)



**Howard Levine** says:

May 11, 2012 at 12:39 pm

The document seems to end on page 35 of 38. Is the pagination in error or are pages missing from the end?

[Reply](#)



**Ashli** says:

May 11, 2012 at 1:34 pm

Hi Howard,

I think your comment was disconnected from the original source. Can you clarify which document you are referring to?

Thanks!

Ashli

[Reply](#)



**Howard Levine** says:

May 13, 2012 at 3:02 pm

The High School Math Scope and Sequence....the bottom of the last page says 35 of 38.



**Bill McCallum** says:

May 16, 2012 at 7:04 am

I've asked Patrick Callahan if he can send me a more complete document.

[Reply](#)



**Ashli** says:

May 16, 2012 at 5:11 pm

Official word from Patrick Callahan is that it's a pagination error.

[Reply](#)



**Joe Ratasky** says:

May 12, 2012 at 11:31 am

Here is a question involving Geometry at the Kindergarten and first grade levels.

My county is currently developing a Scope & Sequence for both grades. During this process, we were tasked with unwrapping the standards and developing unit plans of study. In comparing some of the geometry standards at K and 1, some questions were brought up, mainly involving K.G.1, K.G.2, K.G.4 and 1.G.1. The first grade team assumed that first graders would begin

using defining attributes to determine proper names and classification of shapes, and that kindergarteners would have been recognizing shapes mainly by sight. The kindergarten team felt that kindergarteners would begin identifying, classifying and naming shapes using attributes, mainly number of sides and vertices. The team also felt that many misconceptions would be developed if students were only expected to use visual recognition to identify shapes. We sort of came to an agreement that in K, students might take a single shape and be able to define it based on sides or vertices. “I know this is a triangle because it has 3 sides” Whereas in 1st, students might take the attribute of 3 sides and identify all triangles from a set of shapes. Without the geometry progression documents, is there a better explanation of the difference between these two grade level geometry standards and expectations?

[Reply](#)



**Bill McCallum** says:

May 16, 2012 at 7:21 am

I think having an expectation that a Kindergartner says “I know this is a triangle because it has 3 sides” goes beyond what is called for in the Kindergarten standards. The relevant cluster is:

**K.G. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).**

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

Compare this with

1.G.1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.

It seems to me that the progression is pretty clear here. Of course, as always, that which is not stated is not thereby forbidden; if the definition of a triangle in terms of its attributes comes up naturally in a Kindergarten class, and students seem to be learning from it, then that’s good. But the standards do not require it.

[Reply](#)



**Mark** says:

May 13, 2012 at 9:02 pm

In grade 3 students are to understand the properties of multiplication and the relationship between multiplication and division, 3.OA.5 and 3.OA.6. Regarding Distributive property I’ve seen variations in the interpretations of this in multiple organization and/or state documents – ranging from no appearance of parentheses to beyond what I’m seeing in the standards. 3.OA.7 also includes properties as well. Could you please elaborate on the use of parentheses at this level and clarify these standards.

[Reply](#)



**Bill McCallum** says:

May 16, 2012 at 7:26 am

I think this it a matter of curriculum design exactly when to introduce parentheses. On the one hand, there is no explicit requirement to use parentheses until Grade 5. On the other hand, there's a footnote on 3.OA.8 that says "This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations)." This suggests that parentheses might well be used much earlier. My own preference would be we start to use them when it becomes difficult to say what you want to say without them, and this would probably be in Grade 3.

[Reply](#)



**David Smith** says:

May 14, 2012 at 11:10 am

Bill,

I had a question today from one of our districts about 3.G.2. In the NF domain we restrict denominators to 2, 3, 4, 6, and 8. Does the same restriction apply to partitioning shapes in 3.G.2 or should students be allowed to make as many partitions as they like?

[Reply](#)



**Bill McCallum** says:

May 16, 2012 at 7:29 am

David, it's seems a natural interpretation to apply this restriction to 3.G.2, since part of that standards says "Express the area of each part as a unit fraction of the whole."

[Reply](#)



**Lane** says:

May 16, 2012 at 6:29 am

Regarding proportional reasoning. I understand major US textbook companies have published "CCSS-aligned" textbooks that do proportions the old-fashioned way instead of determining the constant of variation per CCSS. Has anyone seen a textbook that shows the progressions correctly? Does anyone know of a textbook source from one of the model countries (in any language)? We don't have to be able to read the text to follow the examples. Lots of folks are questioning whether the progressions are a new US invention or whether they are truly modeled after a high-performing country's curriculum, so PDers would gain credibility if they had access to an original source. It would be worth the cost, whatever it is, to have an example set of textbooks from, say, Singapore; but again lots of American companies say their textbooks are modeled after Singapore and obviously we can't just take their word for it.

[Reply](#)



**Cathy Kessel** says:

May 17, 2012 at 5:47 pm

Lane, I know of three US sources of textbooks from other countries (which I've listed below). I'm trying to be brief here, but will try to put something more comprehensive on my blog in the next few days.

I think your question splits into three parts:

1. the term “progression.” The terms “learning progression” and (related but not identical) “learning trajectory” seem to be inventions of US mathematics education researchers.
2. the concept of “progression.” in my opinion, the idea of “learning progression” is implicit in textbooks and other curriculum documents from outside the US. For example, check out the discussion of the central sequence of the knowledge package for subtraction with regrouping in Liping Ma’s book *Knowing and Teaching Elementary Mathematics* that starts on p. 15 or for the knowledge package on multidigit multiplication that starts on p. 45 (you can see these via Google Books).
3. the content and order of the CCSS progressions as compared with those of other countries. The CCSS states two types of sources in the list of works consulted. These include: documents of other countries (and analyses of those documents), articles on US research on learning trajectories. So, I think the short answer is yes, the CCSS progressions are a US invention, combining US research and progressions from other countries. However, as the work of Schmidt and others points out, progressions in those other countries aren’t all that different from each other (this of course depends on the level of detail).

Many other countries do not have “standards” but have documents which are more or less comparable with names like “course of study” or “syllabus.” Links to those for several countries (including Japan and Hong Kong) are here: [http://hrd.apec.org/index.php/Mathematics\\_Standards\\_in\\_APEC\\_Economies](http://hrd.apec.org/index.php/Mathematics_Standards_in_APEC_Economies)

You don’t need to pay money for textbooks to get evidence about progressions and the CCSS. There are two types of sources: US research on learning trajectories and documents of other countries (and analyses of those documents). Just to be brief (or at least not incredibly long), I’ll only comment on the latter, but note that both types are listed in the “works consulted” in the CCSS.

Many other countries do not have “standards” but have documents which are more or less comparable with names like “course of study” or “syllabus.” Links to those for several countries (including Japan and Hong Kong) are here: [http://hrd.apec.org/index.php/Mathematics\\_Standards\\_in\\_APEC\\_Economies](http://hrd.apec.org/index.php/Mathematics_Standards_in_APEC_Economies)

I’ve discussed comparisons of CCSS and documents from other countries on my blog: <http://mathedck.wordpress.com/2011/09/06/strange-accounts-of-the-common-core-state-standards/>

Here are the textbook sources that I know. Note that they aren’t necessarily going show things that are identical to what’s in the Progressions (for example, there’s no guarantee that terminology will be identical to the US or even among other countries), but there’s a lot of resemblance).

Singapore Math ([www.singaporemath.com](http://www.singaporemath.com)) has Singapore textbooks (which were originally written for English-speaking Singapore students) adapted for the US. I think this mainly means that the names of things and British spelling and terms (e.g., “ring it” for “circle it”) are adapted to a US audience. I don’t think they have the teachers manuals for the books. (I do, and I find them useful.)

The University of Chicago School Mathematics Project (<http://ucsm.uchicago.edu/resources/translations/>) has translations of Japanese textbooks for grades 7-9 and Russian grades 1-3. (It says that the American Mathematical Society has translations of Japanese textbooks for later grades, but I didn’t see them on the AMS web site and suspect that they may have sold out recently when they were on sale.)

Global Education Resources (GER, <http://www.globaledresources.com/>) has translations of Japanese textbooks for grades 1-6. It’s also got translations of the teaching guides for grades 1-6 and for grades 7-9, and lots of other things,

some of which are free of charge. You can download (free of charge) some translations of the teachers manuals for one textbook series that GER translated from [http://lessonresearch.net/nsf\\_toolkit.html](http://lessonresearch.net/nsf_toolkit.html). These are really nice (I worked on editing the translations) as are the textbooks. You can get a sense of what might be called a “progressions way of thinking” in Learning Across Boundaries: U.S.–Japan Collaboration in Mathematics, Science and Technology Education (free and downloadable at <http://www.lessonresearch.net/LOB1.pdf>). For example, check out the piece that begins on p. 261. In discussing “research lessons” (special lessons created by groups of Japanese teachers as a result of “lesson study”), a Japanese professor of mathematics education says:

A research lesson is only one lesson. However, in doing research lessons we are not thinking about only one lesson. We need to think about the entire unit and how it’s related to other grade levels. That is very important.

He continues (pp. 262-266) by illustrating how that might be done for a lesson on estimating the area of a circle, using excerpts from the GER translations of Japanese textbooks.

Sorry not to put live links, but it is time consuming and I seem not be doing well with this recently (maybe the blog interface has changed).

[Reply](#)



**Lane says:**

May 21, 2012 at 8:54 am

Fantastic. This will give me some additional summer reading!



**June says:**

May 17, 2012 at 11:50 am

A question arose today regarding when to introduce division problems with remainders. Thoughts?

[Reply](#)



**Bill McCallum says:**

May 23, 2012 at 1:45 pm

This happens in Grade 4:

4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

and

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

[Reply](#)



**Howard Levine** says:

May 17, 2012 at 2:51 pm

The The High School Math Scope and Sequence that I've seen using your website has Exponential Functions and Rational and Polynomial Functions in Algebra 2, but New York State has them in Algebra 1. The Common Core isn't so common, anymore!

[Reply](#)



**Bill McCallum** says:

May 23, 2012 at 1:49 pm

The high school standards are not arranged into courses. States wanted this flexibility, so that some could pursue a traditional sequence and some could pursue an integrated sequence. That said, I think you'll begin to find more uniformity once the assessment consortia come out with high school course boundaries.

[Reply](#)



**Rey** says:

May 19, 2012 at 6:03 pm

Hi Dr. McCallum

We hit a snag with this standard:

A-SSE.1 – Interpret expressions that represent a quantity in terms of its context.

Interpret parts of an expression, such as terms, factors, and coefficients.

Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .

Our question is....What does the word “interpret parts of an expression such as factors” here mean? How does one interpret a factor? Does this standard merely mean that we identify parts of an expression? If that is the case, why is the wording used “interpret” instead of “describe” or “identify?”

Thank you 😊

[Reply](#)



**Bill McCallum** says:

May 23, 2012 at 1:57 pm

Note that the standard calls for you to make the interpretation in terms of a context. That's important. Given a naked expression like  $(x + 2)(x - 3)$  there's not much that you can say about the factors except that they are factors. But an expression like  $P(1 + r)$  might arise in a context where an amount of money  $P$  is increased by an interest rate  $r$ . Then the interpretation would go beyond just recognizing  $P$  and  $1 + r$  as factors, to include relating each factor to the context and to the way interest is computed.

[Reply](#)



**Sarah Renninger** says:

May 20, 2012 at 4:44 pm

PARCC's "major clusters" are not the same as the critical areas of the Common Core. I'm confused! Can you help explain?!

Sarah Renninger

Math Coach

New Jersey

[Reply](#)



**Lynn Selking** says:

May 22, 2012 at 9:06 am

Thanks for this wonderful help, Bill.

Regarding K.CC.1 "Count to 100 by ones and by tens," I am wondering if the idea is for the student to be able to speak the two sequences rather than actually count objects. Sometimes, students live through 90 hours of calendar time per year for a number of years and then when asked to count a collection by fives, point to each object and speak a multiple of five. They don't use the sequence to actually organize the objects in the collection and find out how many there are. So instead of 20 objects, they will respond with 100 objects.

It doesn't seem right to expect a kindergarten student to manipulate 100 objects into groups of 10 to count them by tens. Can you give some guidance?

[Reply](#)



**Lynn Selking** says:

May 24, 2012 at 7:50 am

I think as I study a bit more I can answer my own question. Standards 1, 2, and 3 really are about the sequence and standards 4 and 5 are about how many and guidance about the range of number is given.

[Reply](#)



**Brad Burkman** says:

May 23, 2012 at 8:29 am

Bill,

In 5.NF.1, the general formula for fraction addition is given as:

$$a/b + c/d = (ad+bc)/bd$$

I would write it as:

$$(ad+bc)/(bd)$$

I agree that we all know the order of operations that the author intended, but is it clear from what is written, and a good example of how to write fractions?

[Reply](#)



**Bill McCallum** says:

May 23, 2012 at 2:19 pm

Really, it should be  $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ , but the typesetting was limited. I can see your point that there should be been extra parentheses around the  $bd$  when the fraction was shilled.

[Reply](#)



**Lane says:**

May 23, 2012 at 8:34 am

My concern is when do they begin to understand LCD because that memorized formula is a huge stumbling block to Algebra 2 students working with rational expressions.

[Reply](#)



**Bill McCallum says:**

May 23, 2012 at 2:21 pm

Lane, I don't think I understand the question, can you give an example?

[Reply](#)



**Lane says:**

May 23, 2012 at 4:48 pm

In the formula under discussion,  $bd$  is the “easy way” to find a common denominator. However, students who are not fluent in converting to a least common denominator LCD are typically overwhelmed when tasked with solving rational equations. In general, if we have  $x/ab + y/bc = z/ac$  where  $a, b, c$ , are prime polynomials, the LCD is  $abc$ , not  $(abcbad)$ . For instance, if a student needs to solve  $(3y)/(y^2+5y+6) + 2/(y^2+y-2) = (2y-1)/(y^2+2y-3)$  and multiplies all 3 denominators together, then proceeds to multiply the numerators by all the denominators, they end up with high-degree polynomials in the numerator. Yet if the students have been getting away with  $bd$  for years (because their lower level teachers don't know what's ahead) and we try to get them to find LCD, a small war ensues and we are accused of making things more difficult than they need to be.



**Brad Burkman says:**

May 24, 2012 at 7:07 am

Lane,

I'm glad someone else is fighting the good fight. Let's compare strategies for remediation.

Brad, [bburkman@lsmsa.edu](mailto:bburkman@lsmsa.edu).

[Reply](#)



**Lane says:**

May 24, 2012 at 7:42 am

Ha! Sounds like Dr. McCallum will need to start another page for that. I have taught thru Calculus, but prefer analyzing how right-brainers can be taught to “see” what mathees see (hurray Smartboard). A big help is

learning to fluently decompose. Working with high schoolers who have memorized and confused a bunch of rules and steps, I insist they break everything down to primes until they begin to discover the rules on their own: LCD, GCF, working with exponents, factoring out perfect squares from radicals, rationalizing...always with primes until they see. But connections are best built over time, not as a last-ditch effort to save them. I believe CCSS is at least ‘saying’ comprehension is better than memorized formulas. Whether it pans out that way I believe depends largely on this web site.

“5.NF.1, the general formula for fraction addition is given as:  $a/b + c/d = (ad+bc)/bd$ ”  
is another formula to mess up; why not use multiplicative identity on primes til they see?



**Tad W** says:

May 24, 2012 at 10:15 am

I don't teach HS, but I think a natural formula elementary children will derive is  $a/b + c/d = (ad + bc)/(bd)$ . In the way fractions are laid out in the CCSS, the line of reasoning goes something like this:

In Grades 3 & 4, students learned that  $a/b$  is a pieces of  $1/b$  units. So, they find it relatively easy to calculate  $3/5 + 4/5$  because 3  $1/5$  units and 4  $1/5$  units together will be 3+4  $1/5$  units, or  $7/5$ . Then, in Grade 5, when they encounter  $2/3 + 3/4$ , they would say, we can't because 2 and 3 refer to different units. But, they learned in Grades 3 and 4, some fractions may look different but stand for the same numbers – and in Grade 4 they learned how to create equivalent fractions. So, they could say, well,  $2/3$  doesn't always have to look that way (and  $3/4$  can look different, too). So, they find a common unit that can be used to express both  $2/3$  and  $3/4$ . They realize  $1/12$  is an easy option since the way to create equivalent fractions is to multiply both the numerator and the denominator by the same number. So, if you have two unlike denominators, then one easy common unit is to use the product of the denominator as the common units.

I tend to think when students see the need, that's the best time to teach a relevant mathematical idea. If calculation involving rational expressions is where the usefulness of LCD comes in, then maybe that's when the idea should be discussed.



**Lane** says:

May 24, 2012 at 12:16 pm

Your argument is well taken. I would help them see the need for LCD as they need to work with larger denominators. The frustration level we experience with the kids in high school doesn't need to be. I would push for exploration of LCD a year after  $bd$ , beginning with adding fourths to halves with the LCD of 4ths instead of 8ths. Shortly after that time, I believe they should be working with prime factorizations to find the LCD in order to avoid a lot of reducing in the end. If they understand prime factorization, that tool bypasses a lot of mindless “steps” to confuse from Pre-Algebra through Algebra 2.



**Tad W** says:

May 24, 2012 at 10:19 am

I apologize for multiple posts, but I was also wondering about the example Lane gave. After student multiply both sides of the equation with the product of 3 denominators, why do they multiply the terms out to get a higher order equations? Why not factor those expressions that were the denominators? Then, you have some factors that are common in all 3 terms, allowing us to cancel (divide) them out? Then, you will have a quadratic equation at the end. Wouldn't the whole process about equally complicated as finding the LCD?



**Lane** says:



May 24, 2012 at 12:00 pm

In order to cancel a factor from the numerator, of course, it has to be a common factor with the denominator, in essence “1.” In solving rational equations, we sometimes get very messy numerators where we have to 1) combine like terms, 2) factor, 3) cancel. If we have created higher power terms by not getting the LEAST common denominator, the students do not have the tools to easily factor them. They would have factor by finding zeros in their grapher and dividing them out or apply the rational root theorem...very frustrating to throw all that into one problem. If they can find the LCD, then it is very doable.



**Brian Cohen** says:

May 24, 2012 at 11:03 am

I agree with Tad that, “when students see the need, that’s the best time to teach a relevant mathematical idea.” And, the way he talks about introducing the addition of fractions with unlike denominators sounds right to me (with the use of visual fraction models to support understanding of the what’s happening when we multiply both the numerator and the denominator of one fraction each by the same number). But every 4th grade standard related to the addition or subtraction of fractions explicitly states “with like denominators.” Another cluster does require students to find common denominators, but the only application of this that is stated in the 4th grade standards is to compare fractions. The cluster on operations with fractions very clearly states “with like denominators” and, therefore, seems to exclude uncommon denominators.

5.NF.1 (“Add and subtract fractions with unlike denominators...”) is the first standard that explicitly addresses the addition and subtraction of fractions with unlike denominators.

Based on this, it seems to me that the quote from page 10 the Progression for NF that Eric raised (“In Grade 4, students calculate sums of fractions with different denominators...”) would refer to a natural extension of 4th grade standards, but is neither required nor forbidden by the standards themselves.

If this is not the correct interpretation, please advise soon, as I know a LOT of districts that will need to seriously alter plans!

Thanks,  
Brian



**Tad W** says:

May 24, 2012 at 11:28 am

I, too, was surprised when I read the statement mentioned by Eric in the draft progression document. I think it is clear that  $\frac{2}{3} + \frac{3}{4}$  is in Grade 5, but  $\frac{2}{3} + \frac{5}{6}$  will be in Grade 4 according to the progression document but in Grade 5 if we just read the CCSS. I supposed the progression documents are supposed to be an elaboration, maybe the authors of the CCSS meant to include  $\frac{2}{3} + \frac{5}{6}$  in Grade 4.

If you compare  $\frac{2}{3} + \frac{3}{4}$  and  $\frac{2}{3} + \frac{5}{6}$ , perhaps  $\frac{2}{3} + \frac{5}{6}$  is easier procedurally (to find the common unit) but they are equal conceptually (let’s make the unit the same). So, comparing fractions with unlike denominator after learning how to create equivalent fractions (and learning how to add fractions like  $\frac{2}{3} + \frac{5}{6}$ ) but not add/subtract fractions with unlike denominators (where neither is a multiple of the other) is like having to stop an interesting story right before the end and told to wait till next year to finish it — to me.



**Brian Cohen** says:

May 24, 2012 at 11:42 am

lol. Thanks, Tad. Your interpretation is consistent with mine... and your analogy is great!



**Brad Burkman** says:

May 25, 2012 at 10:22 pm

Brian & Tad,

I agree with you both, that “when students see the need, that’s the best time to teach a relevant mathematical idea.” If I read Lane correctly, the concern of high school teachers (including myself) is that fraction addition in CCSS-M seems to stop in fifth grade. In sixth grade students learn to find the lcm, but they don’t apply it to fractions to find the lcd. Then in high school, Tad throws them a sum of rational expressions whose elementary-school equivalent is  $1/6 + 1/8 + 1/9$ . Our students want to use  $6*8*9 = 432$  as the common denominator, when they could use  $72$ . Between fifth and tenth grade, students do not expand their understanding of fraction addition. If in sixth or seventh grade we had them use their new skills in finding least common multiples to subtract

$$7/30 - 5/42 = 7(5*6) - 5(7*6) = (49 - 25)/(5*6*7) = 24/(5*6*7) = 4/35$$

then they’d be ready for Lane’s question in algebra.



**Lane** says:

May 26, 2012 at 7:27 am

Exactly! I’m so glad you succinctly summed it up here. We should be building a continuum, stimulating the “need to know” for the next idea, instead of setting the kids up for a big leap of frustration. Very few kids understand LCD in the abstract sense (rational eqns) if they haven’t already grasped LCD numerically. This is one way we lose a lot of Algebra 2 students. Decomposition is a basic building block for number sense so I don’t understand why CCSS would leave this gap. I think the only way it would make sense if we decided not to teach rational functions.



**Bill McCallum** says:

May 26, 2012 at 8:49 am

I’ve been thinking about how to respond to all this. I was just chatting with my colleague Cody Patterson and he said it in a way that I found very useful: finding least common denominators is a strategy, not an essential component of fraction addition (either numerical or algebraic). It’s important to separate out the task of understanding fraction addition as an operation from the task of finding efficient strategies one might use to find answers in special cases. We don’t want students to think the strategies are the same thing as the operations themselves. Job one is understanding the operation.

Tad summarizes that understanding well: in adding fractions you express each fraction in terms of a common unit. The natural common unit is the unit fraction whose denominator is the product of the denominators of the addends. And this is only common unit that exists in general, so it’s the only one that leads to a general formula. All this is difficult enough conceptually; it muddies the waters to worry about special cases where you might be able to find a smaller common unit. Worse, it might make kids think that somehow finding least common denominators is an essential part of fraction addition (I’ve certainly met students who seem to think that). And, since there is no formula for “adding fractions by finding a least common denominator”, insisting on it gets in the

way arriving at a general formula; the general formula becomes something extra to memorize later, rather than an essential understanding of fraction addition as a general operation. Fraction addition becomes an arcane art. The calculation

$$\frac{1}{6} + \frac{1}{8} + \frac{1}{9} = \frac{174}{432}$$

is 100% mathematically correct. There is not anything even a little bit wrong with it. Once kids have a solid understanding of fraction addition, and if there is time in the curriculum, it is worthwhile pointing out that the answer is equivalent to  $\frac{29}{72}$  and exploring what strategies you might have used to get that answer.

The same comments applies to Lane's original example. One strategy for solving this problem is to multiply both sides by the least common denominator. But as Tad pointed out, another reasonable strategy is to multiply both sides by the product of all the denominators and then cancel common factors from both sides of the equation before expanding. This takes slightly more ink, but provides an opportunity to discuss an important strategy in algebraic manipulation, namely the advantage sometimes of keeping expressions in factored form rather than blindly expanding them out. There are advantages to both strategies, but neither is sacred.

[Reply](#)



**grace says:**

May 23, 2012 at 3:01 pm

It seems my question might have gotten stuck in the moderation queue, so I'm reposting here and would love some advice 😊

Hi! Thanks for providing the opportunity to ask questions about the Common Core Standards. I just wanted to clarify a few things in the high school standards:

- (1) G-GPE.4 Students are asked to "Use coordinates to prove simple geometric theorems algebraically" and the given examples are about proving that four points form a rectangle or that a point lies on a circle. Are there other types of "simple geometric theorems" that students should be familiar with, or guidance I can use to interpret the word "simple"?
- (2) G-GC.2 Similarly, students are asked to "identify and describe relationships..." in circles. Should this include secant theorems, or just the more obvious angle theorems and tangent theorems?
- (3) G-GMD.1 reads "Identify the shapes of two-dimensional cross-sections of three-dimensional objects." Since 7.G.3 reads "Describe the two-dimensional figures that result from slicing three-dimensional figures" and specifies right rectangular prisms and right rectangular pyramids, is it safe to assume that the high school standard includes cross-sections of any and all 3D objects beyond right rectangular prisms and right rectangular pyramids?
- (4) I'm not seeing solving absolute value equations (e.g.  $|x - 3| = 5$ ) in the standards, although I do see absolute value of real numbers in middle school and then graphs of absolute value functions in high school. Is it implied but not explicitly required that students should be able to solve absolute value equations?
- (5) A-APR.3 and F-IF.7 use the language of "when suitable factorizations are available" to describe when students should be finding roots of polynomial and rational functions. Does this mean that students should be comfortable finding factors using GCF, grouping, sums/differences of squares, but not long division? Or not long division with remainders? Where do we draw the line with what is "suitable"?

(6) I see references to the properties of operations (commutative, associative, etc.) in lower elementary standards about addition and multiplication, and in high school standards relating to complex numbers and matrices, but not as they relate to algebraic expressions. Can I interpret this to mean that while students should be familiar with and able to flexibly use the properties, they will not be assessed on being able to name specific properties?

Thank you so much for your time and help!

[Reply](#)



**Eric** says:

May 24, 2012 at 9:40 am

4.NF.3 says, “Understand a fraction  $a/b$  with  $a > 1$  as a sum of fractions  $1/b$ .” This seems to be implying addition of fractions with LIKE denominators. Then reading parts a,b,c,and d, it mentions LIKE denominators, but never UNLIKE denominators. However, when reading the Progression document for NF, it says on pg. 10 under the Grade 5 heading, “In Grade 4, students calculate sums of fractions with different denominators where one denominator is a divisor of the other, so that only one fraction has to be changed.” Then it gives an example of  $1/3 + 1/6$ . In all of the Grade 4 Progression, in the section titled “Adding and subtracting fractions,” it never mentions this idea of adding unlike denominators when one is the divisor of the other. In fact, ALL of the examples show sums and differences with LIKE denominators. Should we be giving 4th grade students problems and questions relating to sums of fractions with UNLIKE denominators. I can see where 4.NF.3a could lend itself to this type of thinking, but with the root standard defining it as “sums of fractions  $1/b$ ,” this seems to be contradictory.

[Reply](#)



**Julie** says:

May 25, 2012 at 10:42 am

Hello! I just went through my first year of trying to implement the Common Core Math Standards for 8th grade and I do not feel like I did a great job. I just watched Phil Daro’s video about how we teach to get answers instead of the math. I want to be that teacher that lets the students explore the math instead of me telling them how to get there. Do you know where I can find learning tasks, messy problems, thinking problems, what ever you want to call them, for 8th grade common core so the students in my class don’t worry about answers and worry about the process? I know PARCC will be putting out sample tasks this summer, but I would love to start working on things NOW! 😊 I don’t feel like I can “create” problems myself that are not a traditional type of story problem.

Thank you for your time,

Julie Brandolino

[Reply](#)

