Facilitator Notes

One entry activity (or ice breaker) for the topic of Exponential Growth might be::

- Have one students group begin with 1 M&M
- Every 2 seconds double that amount.
- Another group also begins with 1 M&M. Every 10 seconds we will quadruple the amount.
- Have students/teachers discuss the results.
- Then change initial amounts. Have students/teachers consider the implications of changes made and discuss other changes that can be made—predicting the results and then judging their predications against the actual results.

This activity can be varied in many ways to explore different aspects of exponential functions.
The initial slides are overview slides that provide a common language for the Common Core State Standards for Mathematics at the High School Grade levels.

The graphics came from the Arizona Department of Education Overview document (found on the Standards and Assessment webpage at azed.gov)

The Standards targeted by the professional development include 2 domains from the conceptual category, Functions. The domains are Interpreting Functions and Linear, Quadratic, & Exponential Models. There are also Mathematical Practices that would most likely be observed (but are not required).

*Note the parenthetical that states the practices are *not* required, as the Mathematical Practices should be observed by teachers but not dictated to students.

*Also note that MP1 is typically seen in all problem solving, and was therefore not specifically noted. The standards were chosen because they look like standards many states already have at this course level, but have nuances that require the student to have a deeper understanding than the ability to flexibly manipulate algebraic expressions.

There are also connections to other standards in CCSS, but the listed standards are the *target* standards.

HS.F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context

HS.F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to y. interpret expressions for exponential functions. For example, identify percent rate of change in functions such as

y = (1.02)2, y = (0.97)t, y = (1.01)12t, y = (1.2), $y = (1.2)^{\frac{t}{10}}$ and classify them as representing exponential growth or decay.

MP.3 Construct viable arguments and critique the reasoning of others
MP.4 Model with mathematics
MP.7 Look for and make use of structure

The comparison between Israel and Iraq is designed to help students attend to structure.

- Israel will never catch up!
- Ideally students will set up equivalent expressions and realize that Istrael's population will never catch up to Iraq' because of the lower annual growth rate
- Israel: 1.5% annual growth rate Iraq 2.3% annual growth rate
- Initially students will need to ask themselves, "What form of the equation should I use?"
- They may anticipate asking, "What does that look like graphically? algebraically?" but then realize once they have made their expressions comparable that they do not need to go any further.

The Facebook investigation is designed to be less explicit and allow for more discussion and decision. Students may consider the Facebook information in terms of a linear model or an exponential model. This will allow for discussion of which model makes sense given what students know about Facebook. The model can then be reconsidered once students begin to learn about Logarithmic growth to reason about whether that model would better model Facebook growth.
Students will need to think about the structure to use in order to graph both on the same axes, that is students must use equivalent expressions *purposefully—attending to structure.*

Students may even discuss that with businesses and schools having Facebook pages, is it reasonable the the number of "users" could exceed he Earth's population, and if so what is an appropriate definition of "user"?

The big idea: The structure of the model should follow from the context, rather than force all contexts to fit into the same structure.

After the PowerPoint is completed:

Given extra time, teachers can be taken to Iluminations.nctm.org.
Teachers can open the investigation, Drug Filtering.
This is a typical investigation (pre-CCSSM).

Ask the teachers: Will this take us as far as we need to go with students?
Encourage teachers to include a second page to the investigation.

Some follow up questions on the second page may include:

- How can we describe this relationship algebraically?
- Can we rewrite this into a form that shows the hourly decay rate? Why would that be advantageous?
- What if a child were to be given this medication? What could we reasonably say about the rate of metabolism? How would that impact our algebraic model?
- What if the initial dose was doubled? What would change in our model? What remains invariant? How would this transform the graphical model?

*Please note that *invariant* was chosen as it relates to Driscoll's Algebraic Habits of Mind (Driscoll, 1999).

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