

Knowledge of Mathematics for Teaching: Some Thoughts from Mathematicians

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Mathematical Experience

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What do secondary mathematics teachers need to know about mathematics in order to be effective educators and leaders in their school environments?

Teachers need to know the math they teach – and more!

“Teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach.”

– Task force on Teachers and Teacher Preparation, National Mathematics Panel 2008.

Teaching as an apprenticeship to a life of exploration:

... we thought of teaching as an art of developing in the very young curiosity, a sense of adventure, and a sensitivity of perception, as well as the mastery of basic skills. ... the teacher's **mastery of the art of teaching ... is certainly not an obvious consequence of familiarity with factual information.** ... the teacher's performance reflected [his/her] whole intellectual outlook.

Arnold Ross

Knowing Mathematics as a Mathematician

- ▶ Examining the whole mathematical enterprise
 - ▶ as a coherent body of knowledge
 - ▶ as a way of thinking and inquiring about the world we live in
- ▶ Having *personal experience of the doing of mathematics*:
 - ▶ asking questions
 - ▶ grappling with problems
 - ▶ building intuition
 - ▶ developing theories for oneself
 - ▶ becoming completely absorbed in mathematical activity for a sustained period of time

Experience first:

It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student understand. By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing.

Arnold Ross

Mathematical Habits of Mind

- ▶ **Acquiring experience**
 - ▶ numerical experimentation and alert observation
 - ▶ mathematics as an empirical science
 - ▶ practice – enhancing skills
 - ▶ inductive reasoning – building intuition and sense-making
- ▶ **Use of language**
 - ▶ precision
 - ▶ asking good questions, formulating conjectures
 - ▶ reasoning – proofs and disproofs
- ▶ **Review**
 - ▶ identifying important ideas
 - ▶ making sense of complex problems
 - ▶ looking for connections
- ▶ **Generalization**
 - ▶ broadening applicability
 - ▶ questioning answers

Beliefs about the nature of mathematics

- ▶ **Mathematics is natural**
 - ▶ The empirical nature of mathematics
 - ▶ People do mathematics naturally
- ▶ **Mathematics exists independent of us**
 - ▶ We can perform experiments
 - ▶ We can test ideas and decide for ourselves
- ▶ **Experience precedes formality**
 - ▶ “Meaning” is determined by experience
 - ▶ Definitions and theorems are capstones
 - ▶ Language is a tool for coming to terms with experience
- ▶ **Mathematics is the study of structure**
 - ▶ Operations, order
 - ▶ Shape
 - ▶ Continuity
 - ▶ Transformation
- ▶ **Mathematics is the art of figuring things out**

Contexts for Experience

- ▶ Immersion Experiences
 - ▶ Summer institutes
 - ▶ Teacher Education Programs
 - ▶ Study Groups
- ▶ Classroom Discussions (Lesson study / analysis of classroom animations)
- ▶ Curricular Materials
- ▶ Using technology
- ▶ Independent Problem-Solving
- ▶ Everyday Life

Role of Experience

- ▶ People learn naturally from experience
- ▶ Low threshold high ceiling
- ▶ It's democratic – everyone has had experiences of their own – teachers and students alike
- ▶ Meaning is *always* anchored in Experience
- ▶ Making sense of things – building intuition
- ▶ Noticing connections
- ▶ Expectation of structure
- ▶ Finding proofs
- ▶ Making definitions
- ▶ The process of abstraction –condensation of experience
- ▶ Experience and understanding experience is what mathematics is all about

- ▶ What else is needed to help teachers bring what they know to their students?
 - ▶ Authentic experiences of doing mathematics can have dramatic impact on the way teachers use and think about mathematics **for themselves**;
 - ▶ For some teachers this transfers easily and naturally to the experiences they design for their students.
 - ▶ But this transfer is not automatic for all teachers, even in cases where they have changed the way they do mathematics for themselves.
 - ▶ Beliefs about students?
 - ▶ Beliefs about how students learn?
 - ▶ Other competing obligations?

Thank You!

THANK YOU!