

FOCUS ON MATHEMATICS: RESEARCH

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March 25, 2011

The Focus on Mathematics Approach

The *Focus on Mathematics* approach involves teachers, mathematicians, and mathematics educators, all working in programs that put mathematics at the core of a tightly connected set of professional development activities, designed to promote teachers' mathematical knowledge for teaching.

WHAT WE'RE **NOT** TRYING TO DO

We're **not** trying to . . .

- “prove” something,
- evaluate teachers.

Instead, we're trying to **learn** for ourselves (and the field).

FoM FRAMEWORK FOR MKT

The FoM team uses four large and overlapping categories to characterize some of the ways in which teachers know and understand mathematics:

- 1 As a scholar
- 2 As an educator
- 3 As a mathematician
- 4 As a teacher

Note: See handout for more details.

WORKING DEFINITION OF MHoM

We define *mathematical habits of mind* (MHoM) to be the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.

RESEARCH QUESTION

What are the mathematical habits of mind that secondary teacher use, how do they use them, and how can we measure them?

MEASURING TEACHERS' MHoM

- What “habits” should we measure?
 - Emphasis on those MHoM that teachers actually use in their professional work.
- How should we measure them?
 - Paper and pencil assessment? (Pilot of draft in 2010)
 - Think aloud? (Work in progress)
 - Classroom observation protocol/“diagram”? (Also in progress)

NO RODS OF LENGTH 1

You can use “rods” of integer sizes to build “trains” that all have a common length. For instance, a “train of length 8” is a row of rods whose combined length is 8. Here are some examples:

2	3	3
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3	2	3
---	---	---

1	1	5	1
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1	1	1	3	2
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7	1
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8

Question: How many trains of length 10 can you make with *no* rods of length 1?

NO RODS OF LENGTH 1

Some food for thought:

- How did you approach the problem?
- What tools did you bring to bear as you worked on it?

MATHEMATICIAN DATA

Todd Abel, yesterday: “What exactly do you mean by ‘doing mathematics the way mathematicians do’?”

- Preliminary **think-aloud** problems in which mathematicians solve unfamiliar problems and then reflect on the what tools they brought to bear on solving them.
- Preliminary **survey** to try to understand what habits mathematicians deem important in their professional work. (Obtained some preliminary data and feedback from approximately 40 “Friends & Family” mathematicians.)

TEACHER DATA

We have three complementary ways of collecting data from secondary teachers:

- **Think-aloud:** What does the teacher “know” about mathematics?
- **Survey:** What does the teacher “value” in doing and teaching mathematics?
- **Classroom observation:** What does the teacher “execute” in the classroom (from the MHoM point of view)?

FOCUS ON MHoM

In collecting teacher data, we have narrowed to the following MHoM categories:

- Performing experiments
- Looking for structure
- Using precise language

THE CASE OF MR. HARLESS

This is Mr. Harless, a private school mathematics teacher. He has been teaching for eight years and doing PROMYS for five.



MR. HARLESS'S THINK-ALOUD: EXPERIMENTATION

Working on the “No Rods of Length 1” problem, Mr. Harless’s initial approach was to consider smaller cases:

- “I see ten and my first thought is definitely **not** to try to understand trains of length ten. I’d rather understand smaller ones [so] that I get some **intuition** and **concrete data** to go with the problem and hopefully find a **pattern**.”
- “I think that solving math problems is hard enough, so I should try to understand the smallest, simplest, most basic case as well as I can and build from there and try to understand what’s going on.”

MR. HARLESS'S THINK-ALOUD: STRUCTURE

Mr. Harless readily recognized the Fibonacci pattern and spent most of his time trying to **justify** this underlying structure:

- “To get some satisfaction out of this [problem], there has to be some sort of structure rather than just a final number of how many trains there are.”
- “I want to see what’s going on and understand and make sense of what I’m seeing. So I don’t have much interest in the actual numbers, I want to see how they’re related to each other.”

MR. HARLESS'S THINK-ALOUD: LANGUAGE

During his problem-solving, Mr. Harless repeatedly said to himself, “I just need to be more precise in my language”:

- “If I can’t write it down clearly, then I probably don’t really understand it. So partly saying *I need to write this down precisely* is code for saying *I don’t understand this completely. I need to see what’s going on.*”
- “I mean, there’s just no way to evaluate something that’s a vague statement. If I’m going to work to say whether I think my conjecture is true or false, I need to have a precise conjecture to work with.”

MR. HARLESS'S CLASSROOM OBSERVATION

Warm-up problem.

A function follows the rule below for integer valued inputs:

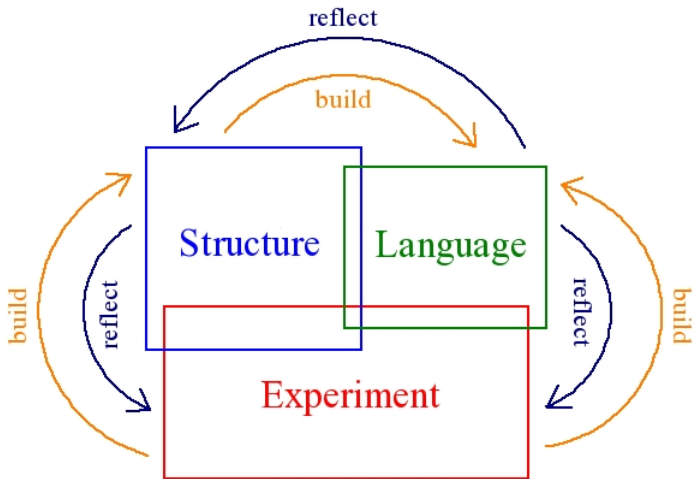
The output for a given input is $\frac{3}{2}$ greater than the previous output.

Make a table that matches the description. Can you make more than one table?

MR. HARLESS'S CLASSROOM OBSERVATION

What MHoM did Mr. Harless use in his classroom instruction?

PRELIMINARY DIAGRAM FOR MR. HARLESS'S CLASS



MR. HARLESS'S SURVEY

When we surveyed Mr. Harless, he selected certain habits as most valuable in his professional work:

- Construct one's own data.
- Discover the structure (that is not apparent at first) by experimenting and seeking regularity and/or coherence.
- Write and speak in sentences having precise meanings that can be tested for validity.

THE TRIANGLE OF DATA

Patterns emerge in the teachers' solving math problems, in their surveys, and in their teaching.

We're missing plenty here with this "triangle of data." It's not "predictive" – for example, it's clear from the data that teachers' beliefs about students and how they learn plays a big role in how the teachers' own habits are visible in his or her classroom. But what we see in the teachers' classrooms is evocative of what we see in their surveys and think alouds.

THREE BELIEFS AND THREE QUESTIONS

We believe there are choices Mr. Harless makes that would not be available to him if he did not know mathematics in the ways he knows it. But what is it, precisely, about how he knows mathematics that allows him to make those choices?

We believe Mr. Harless exhibits tremendous coherence in what he “knows” about mathematics, what he values, and what he executes in his classroom. What is it about Mr. Harless and his context that allow him to be so consistent?

We believe that programs like PROMYS, PCMI, and FoM have an affect on teachers’ knowledge, values, and execution. What factors affect their impact?