Mapping the AP / College Calculus Transition

IM&E Workshop, March 27–29, 2010

Martina Bode (Northwestern University), Gabriel Kortuem (Faribault High School, MN), Gavin LaRose (University of Michigan), Joceline Lega (University of Arizona), Martin Sade (Sonora Science Academy, AZ)

1 Introduction

The most recent (2004) data from the U.S. Department of Education states that many students entering a college calculus course have taken calculus in high school, the majority in Advanced Placement (AP) courses. Therefore, students enrolling in college calculus courses have a wide range of background preparation, from weak precalculus to AP credit that allows them to place into calculus II or III. In this document, we consider the transition of students from AP courses to college calculus. There are a number of studies that suggest that AP classes prepare students well for college classes, especially those students who place into calculus II and calculus III [1, 2, 3]. However, students who do not take the AP exam, or whose high school calculus is weak, may have difficulty in their college calculus course. Therefore, we focus here on students making the transition from high school AP calculus classes to calculus I in college, as opposed to students starting in collegiate calculus II or III. Our goals are therefore

1. to provide insight into the nature of these transitional difficulties,
2. to provide information and resources to teachers on both sides of this transition that may mitigate students' difficulties, and
3. to encourage conversations between high school and college calculus teachers to increase students' success at bridging the transition.

The intended audience for this document includes both instructors of college-level calculus courses and high school teachers of AP calculus. With a better understanding of the students who are coming into college classes from AP classes, college faculty have a better chance of retaining them as math students and math majors. Similarly, by better understanding the nature and demands imposed by the college classes into which their students are going, high school teachers have a better chance of enhancing those students' success.

2 Comparing AP and college calculus

Because the difficulties that we are seeking to address in this document center around students' transition from high school AP calculus courses to college calculus I, an understanding of the essential characteristics of both of these courses is valuable. The lists below illustrate some of the key differences and similarities of these courses.

A high school AP Calculus AB course:

- usually meets five times per week over a year (almost four times the class time of a college course),
- covers slightly more material than a college calculus I course,
- usually has five to thirty students per class,
- may use collaborative work, small groups, projects, *etc.*, in addition to lecture,
- will require graphing calculators in class,
- always assigns and grades homework,

- almost always has support to catch and help students who get lost or behind,
- provides multiple, different assessments and provides very frequent feedback to students and their parents as to how students are performing in the class,
- has tests that include multiple choice, free response, and open-ended items,
- focuses on preparing students for the AP exam, and is therefore a a terminal course at the high school level, and
- may enroll students who have weak pre-calculus skills (trigonometry, logarithms/exponents, algebra).

A college calculus I course:

- usually meets three times per week, possibly with a weekly recitation,
- covers the material in a high school AP Calculus AB course,
- depending on the college or university, may be a lecture of hundreds of students, or may be a small section,
- rarely includes interactive or cooperative work in class,
- may use graphing calculators, but this varies widely by institution,
- often uses online homework, if there is any daily homework,
- is composed of students who are more often than not learning the culture of college,
- often has support structures to help students who get lost or behind, but in general students must actively seek out these resources for themselves,
- provides fairly infrequent feedback to students as to how they are performing in the class,
- may have multiple choice, free response, or other types of problems on tests depending on the institution,
- is a course intended to lead to more mathematics and science courses, and
- may enroll students who have weak pre-calculus skills (trigonometry, logarithms/exponents, algebra).

3 S

uggestions and resources for AP calculus teachers
To successfully teach an AP calculus course, it is not only essential to understand the problem types and topics emphasized by the AP test, but also to have a deep understanding of calculus itself. This understanding may be obtained in many ways, including:

- taking the sample AP test, which is available at the AP Central site cited in the "Additional resources" section at the end of this document. The practice exam will also help you to take note of any material you need to review or re-learn prior to teaching.
- exploring the AP Central website, which provides a wealth of information, including scoring rubrics, exemplary problem solutions and sample responses with scoring commentary, and information (under "professional development") on workshops or summer institutes which may aid in instructional design choices,

- communicating with previous teachers of AP calculus, as well as current AP calculus teachers. Joining the AP Calculus Forum hosted by Drexel University is one way to do this. Information on joining the forum can be found in the resources below.
- being aware of the published study and review guides for the AP test, which can be used to aid both you and your students in teaching and learning calculus. Barron's and Princeton Review are two examples of such review guides.

When teaching AP calculus it is essential to prepare students for the AP test. However, because some of the students in AP calculus will subsequently enroll in college calculus (and other mathematics) courses, the course should also have the goal of preparing students for those courses. This may be accomplished in most cases by working to

- promote problem solving skills and serious mathematical thinking,
- incorporate applications throughout the course, such as utilizing related rates, optimization, and accumulation problems as new function families are introduced in the course,
- develop students' skills at metacognition. This can be done by providing individual learning inventories so that students gain an understanding of their own strengths and weaknesses in acquiring new knowledge and skills, and
- promote a realistic understanding of the expectations of the college culture. This may be accomplished by having "college-level" expectations in your high school course, and may be reinforced by having former students talk to your class about expectations in their college courses (including grading, quality of work, *etc.*), the structural differences between their college and high school courses (including use of calculators and other technology, homework, *etc.*), and the study and time management skills needed in the college environment.

Sample problems that illustrate the type of problem solving, mathematical understanding, and application suggested above may be found in the resources section at the end of this pamphlet.

4 Suggestions and resources for college calculus teachers

Students in college calculus I courses have a wide range of background and mathematical skill in both pre-calculus and calculus material. Some students enter the college calculus classroom having had an AP course, at the end of which they took the AP exam and scored well or moderately well, some with an AP course after which they did not take the AP exam, some with a calculus course that that was not an AP course at all, and some with no calculus background at all (and in some cases a tenuous precalculus background as well). The calculus background of the students from AP courses will be generally similar, but the same is not guaranteed of other calculus courses. As a preliminary step to understanding the background with which students who took AP calculus enter their college classrooms, we encourage college instructors to take the AP sample test available at the AP Central site indicated in the resources section below.

Because of the high percentage of college calculus students who have had high school calculus, one challenge in the college calculus classroom is that of teaching students calculus material for the second time, as well as teaching students with widely varying understanding (and self-awareness of their understanding) of the mathematics that it entails. This challenge is compounded by many students also being new to college or university and having to work through the different expectations and requirements implicit in this new academic environment.

Specific techniques and ideas that may be appropriate to help address these problems include:

- selecting a course syllabus that is constructed to reflect the backgrounds of the students in the course,

- promoting to students the availability of support structures on campus such as tutoring centers and resources to develop missing prerequisite knowledge,
- ensuring that the expectations and requirements of the course are clear,
- teaching material in a manner that develops students' deeper mathematical intuition, and
- developing students' metacognition skills and understanding of what they know and how they are thinking about it.

We discuss each of these below.

In many cases the course syllabus may be fixed (*e.g.*, in cases where the syllabus is determined by a course coordinator or client departments). However, when it is possible to tailor a syllabus to spend less time on material that is review for the students in a particular class, and more time on topics with which students struggle, it may allow both students who are new to the material the opportunity to assimilate it and students who have seen it before the time to assess their preexisting knowledge and develop a deeper understanding.

A major hurdle for students who are new to the college and university environment is adjusting to the lack of explicit or immediate feedback and support triggered by their performance. Thus, actively publicizing the existing support structures, such as math tutoring centers and online skills development modules, that are already available on campus may be necessary in order for students to be aware of and use these resources.

As noted in Section II, the grading and assessment in a college course are frequently very different from those in high school AP courses. The use of technology and calculators and requirements for study time and independent work can be vastly different as well. Explicitly noting the characteristics of the college course (in course syllabi, websites, *etc.*) may help address the change in expectations and requirements that students experience when making the transition from an AP to a college course.

While students enter their college calculus course with a wide variety of differing backgrounds, all students benefit from the development of skills and conceptual understanding that allow them to learn and engage with the material. There are a number of mechanisms that may be used to promote this, including the active use of problem solving and higher-order mathematical thinking skills, as well as development of students' metacognition skills. Methods for accomplishing these aims include the use of interesting problems, similar to the example problems given in the resources section at the end of this document, and the use of group work, peer instruction, discussion, and evaluation in class.

5 Communication between high school and college calculus teachers

The efforts of high school and college calculus instructors to address the challenges faced by students making the transition from high school to college calculus will be far more effective if these two groups of instructors collaborate and discuss their teaching experiences, rather than proceeding independently. Such communication may take many forms, from an informal conversation (which may be initiated by either instructor) to a more formal context such as a teacher circle or discussion group (which should in most cases originate with the college instructor).

While it is beyond the scope of this document to prescribe how such a communication should be initiated or maintained in every situation, there are many resources and ideas that may generally serve to facilitate the development of such ongoing communication, including

- outreach to education schools and departments at local collges and universities,
- GK-12 programs (see http://www.nsfgk12.org/),
- math circles (see http://www.mathcircles.org/),
- high school teacher circles (see http://www.mathteacherscircle.org/),
- MAA sessions at section and national meetings relevant to AP calculus teachers and courses
- AP summer institutes, and
- the AP calculus forum hosted by Drexel University (http://apcentral.collegeboard.com/apc/public/homepage/4631.html).

We encourage teachers of mathematics to consider and investigate these as appropriate in their local environment.

6 Additional resources

The College Board's AP calculus website is the best place to start for resources about the AP tests and AP course content. This is http://apcentral.collegeboard.com/, and includes

- sample AP tests, grading rubrics, and sample student solutions,
- sample AP syllabi (http://apcentral.collegeboard.com/apc/public/courses/syllabi/index.html),
- Calculus AB/BC Course Descriptions (http://www.collegeboard.com/student/testing/ap/sub_calab.html),

and more. In addition to these, sample syllabi and course expectations for college courses may be found on many college and university calculus webpages; for example,

- The University of Arizona: http://math.arizona.edu/~calc/,
- The University of Maryland: http://www.math.umd.edu/undergraduate/courses/dcp/courseMATH140.html,
- The University of Michigan: http://www.math.lsa.umich.edu/courses/115/,
- The University of Texas: http://www.ma.utexas.edu/academics/courses/m408k/.

Finally, sample problems that may require student problem solving and deeper understanding of the mathematical concepts they treat may be found in the *Problem Solving* pamphlet in the Mapping Calculus group of pamphlets.

References

[1] Morgan, R. and L. Ramist. 1998. *Advanced Placement Students in College: An Investigation of Course Grades at 21 Colleges.* Educational Testing Survey Report No. SR-98-13. Princeton, NJ. http://www.collegeboard.com/press/releases/50405.html

[2] Morgan, R. and J. Klaric. 2007. *AP(R) Students in College: An Analysis of Five-Year Academic Careers.* College Board Research Report No. 2007-4. New York. http://professionals.collegeboard.com/data-reports-research/cb/title

[3] Keng, L. and B.G. Dodd. 2008. *An Investigation of College Performance of AP and Non-AP Student Groups.* http://professionals.collegeboard.com/data-reports-research/cb/title