Test 1 | **Linear Algebra** | **May 31, 2019**

MATH 413/513

Dr. Abdul-Rahman

Name: ________________________ **Signature:** ________________________

Solve <u>Seven</u> out the following <u>Eight</u> questions.
Show all your work!

1. [15 points] Suppose that (v_1, v_2, v_3, v_4) is a basis for a vector space V. Prove that

$$(v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4)$$

is also a basis for V.

2. [15 points] Suppose $p_0, p_1, \ldots, p_m$ are polynomials in $\mathbb{F}_m[z]$ such that $p_j(5) = 0$ for each j. Prove that $p_0, p_1, \ldots, p_m$ is linearly dependent in $\mathbb{F}_m[z]$.

3. [15 points] Suppose that U and W are subspaces of $\mathbb{R}^8$ such that $\dim U = 3$, and $\dim W = 5$ and $U + W = \mathbb{R}^8$. Prove that $\mathbb{R}^8 = U \oplus W$.

4. [15 points] Prove or give a counterexample: Let U_1, U_2 and U_3 are subspaces of the finite dimensional vector space V. If $V = U_1 + U_2 + U_3$ and

$$U_1 \cap U_2 = U_1 \cap U_3 = U_2 \cap U_3 = U_1 \cap U_2 \cap U_3 = \{0\}$$

then $V = U_1 \oplus U_2 \oplus U_3$.

5. [15 points]

   (a) Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under addition but not a subspace of $\mathbb{R}^2$.

   (a) Give an example of a nonempty subset U of $\mathbb{R}^2$ such that U is closed under scaler multiplication but not a subspace of $\mathbb{R}^2$.

6. [15 points] Let $U = \{p \in \mathbb{F}_4[z];\ p(2) = p(5) = 0\}$.

(a) Prove that U is a subspace of $\mathbb{F}_4[z]$.

(b) Find a basis for U.

(c) Find a subspace W of $\mathbb{F}_4[z]$ such that $\mathbb{F}_4[z] = U \oplus W$.

7. [15 points] Consider the subset of $\mathbb{R}^4$, $U = \{(x_1, x_2, x_3, x_4) |\ x_4 = x_2 + x_3\}$.

(a) Show that U is subspace of $\mathbb{R}^4$.

(b) Find a basis for U.

(c) Find a subspace W of $\mathbb{R}^4$ such that $\mathbb{R}^4 = U \oplus W$.

8. [15 points] Consider the vector space $\mathbb{R}^{[0,1]}$ of functions $f : [0,1] \to \mathbb{R}$. For $b \in \mathbb{R}$ let

$$S_b := \left\{ f \in \mathbb{R}^{[0,1]} \mid f \text{ is continuous on } [0,1] \text{ such that } \int_0^1 f(x)dx = b \right\}$$

Show that S_b is a subspace of $\mathbb{R}^{[0,1]}$ if and only if $b = 0$.