MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 8

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SUMMER 2019

- **Submit questions 2 and 5 for grading.**
- **<u>Due on:</u> Tuesday June 25, 2019.**
- **You will have a quiz from the following questions on Tuesday June 25, 2019.**

(1) Suppose $(V_1, \langle \cdot, \cdot \rangle_1), \ldots, (V_n, \langle \cdot, \cdot \rangle_n)$ are inner product spaces. Show that

$$\langle (u_, \ldots, u_n), (v_1, \ldots, v_n) \rangle := \langle u_1, v_1 \rangle_1 + \ldots + \langle u_n, v_n \rangle_n$$

defines an inner product on $V_1 \times \ldots \times V_m$.

(2) Let (e_1, e_2, e_3) be the canonical basis of $\mathbb{R}^3$, and define

$$f_1 = e_1 + e_2 + e_3, \quad f_2 = e_2 + e_3, \quad f_3 = e_3$$

(a) Apply the Gram-Schmidt process to the basis (f_1, f_2, f_3).
(b) What do you obtain if you instead applied the Gram-Schmidt process to the basis (f_3, f_2, f_1)?

(3) Let $\mathbb{R}_2[x]$ be the inner product space of polynomials over $\mathbb{R}$ having degree at most two, with inner product given by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)\ dx, \text{ for every } f, g \in \mathbb{R}_2[x].$$

Apply the Gram-Schmidt procedure to the standard basis $\{1, x, x^2\}$ for $\mathbb{R}_2[x]$ in order to produce an orthonormal basis for $\mathbb{R}_2[x]$.

(4) Prove that

$$16 \leq (a+b+c+d)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}\right)$$

for all positive numbers a, b, c, d.

(5) Let $n \in \mathbb{Z}_+$, and let $a_1, a_2, \ldots, a_n, b_1, \ldots, b_n \in \mathbb{R}$ be any collection of $2n$ real numbers. Prove that

$$\left(\sum_{k=1}^{n} a_k b_k\right)^2 \leq \left(\sum_{k=1}^{n} k a_k^2\right)\left(\sum_{k=1}^{n} \frac{b_k^2}{k}\right).$$

(6) Let V be a finite dimensional vector inner product space over $\mathbb{F}$, and suppose that $P \in \mathcal{L}(V)$ with $P^2 = P$ and $\text{null}(P) = (\text{range}(P))^{\perp}$. Prove that P is an orthogonal projection.
Hint: compare and use Question 1 from Hw 7.

(7) Suppose V is finite dimensional and U is a subspace of V. Let P_U be the orthogonal projection onto U. Show that

$$P_{U^\perp} = \mathbb{1} - P_U$$

where $\mathbb{1} \in \mathcal{L}(V)$ is the identity map on V.