

**MATH 413/513 (LINEAR ALGEBRA)**  
**HOMEWORK 3**

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SUMMER 2019

- **Submit questions 4 and 10 for grading.**
- **Due on: Wednesday May 29, 2019.**
- **You will have a quiz from the following questions on Wednesday May 29, 2019.**

(1) Show that the vectors  $v_1 = (1, 1, 1)$ ,  $v_2 = (1, 2, 3)$ , and  $v_3 = (2, -1, 1)$  are linearly independent in  $\mathbb{R}^3$ . Write  $v = (1, -2, 5)$  as a linear combination of  $v_1, v_2$ , and  $v_3$ .

(2) Define  $U = \text{span}(u_1, u_2, \dots, u_n)$ . Suppose  $v \in U$ , prove

$$U = \text{span}(v, u_1, u_2, \dots, u_n).$$

(3) Suppose that  $v_1, v_2, \dots, v_n$  is a linearly independent set of vectors in  $V$ . Given any  $w \in V$  such that the set of vectors

$$v_1 + w, v_2 + w, \dots, v_n + w$$

is linearly dependent, prove that  $w \in \text{span}(v_1, v_2, \dots, v_n)$ .

(4) Suppose  $v_1, v_2, v_3, v_4$  is a basis of  $V$ . Prove that

$$v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4$$

is also a basis of  $V$ .

(5) Find the dimension of the following subspace of  $\mathbb{R}^4$

$$U = \{(x_1, x_2, x_3, x_4) \mid x_4 = x_1 + x_2\}.$$

(6) Let  $\dim(V) = n$  for some  $n \in \mathbb{Z}_+$ . Prove that there are  $n$  one-dimensional subspaces  $U_1, U_2, \dots, U_n$  of  $V$  such that

$$V = U_1 \oplus U_2 \oplus \dots \oplus U_n.$$

(7) Let  $U = \{p \in \mathbb{F}_4[z] : p(6) = 0\}$ .

(a) Find a basis of  $U$ .

(b) Extend the basis in part (a) to a basis of  $\mathbb{F}_4[z]$ .

(c) Find a subspace  $W$  of  $\mathbb{F}_4[z]$  such that  $\mathbb{F}_4[z] = U \oplus W$ .

(8) Let  $U$  and  $V$  be four-dimensional subspace of  $\mathbb{R}^7$ . Prove that  $U \cap V \neq \{0\}$ .

(9) Suppose  $p_0, p_1, \dots, p_m \in \mathbb{F}[z]$  are such that each  $p_j$  has degree  $j$ . Prove that  $p_0, p_1, \dots, p_m$  is a basis of  $\mathbb{F}_m[z]$ .

(10) Suppose that  $p_0, p_1, \dots, p_m \in \mathbb{F}_m[z]$  satisfy  $p_j(1) = 0$ . Prove that  $(p_0, p_1, \dots, p_m)$  is a linearly dependent list of vectors in  $\mathbb{F}_m[z]$ .