# MATH 413/513 (LINEAR ALGEBRA)
## HOMEWORK 2

DR. ABDUL-RAHMAN

SUMMER 2019

- **Submit questions 2 and 7 for grading.**
- **Due on: Friday 05-24-2019.**
- **You will have a quiz from the following questions on Monday 05-20-2019.**

(1) The empty set fails to satisfy only one of the requirements of Vector Spaces. Which one?

(2) Let $\mathbb{F}[z]$ be the vector space of all polynomials with coefficients from the field $\mathbb{F}$. Let
$$W = \mathbb{F}[z] \times \mathbb{F}^2 = \{(q, u);\ q \in \mathbb{F}[z], u \in \mathbb{F}^2\}.$$
Equip $W$ with a vector addition $(+)$ and a scalar multiplication $(\cdot)$ in such a way that $W$ becomes a vector space, and prove that $W(+, \cdot)$ is a vector space over $\mathbb{F}$.

(3) Suppose $U_1$ and $U_2$ are subspaces of $V$. Prove that the intersection $U_1 \cap U_2$ is a subspace of $V$.

(4) Prove that the union of two subspaces of $V$ is a subspace of $V$ if and only if one of the subspaces in contained in the other.

(5) Prove or give a counterexample: if $U_1$, $U_2$, and $W$ are subspaces of $V$ such that
$$U_1 + W = U_2 + W$$
Then $U_1 = U_2$.

(6) Prove or give a counterexample: if $U_1$, $U_2$, and $W$ are subspaces of $V$ such that
$$U_1 \oplus W = U_2 \oplus W$$
Then $U_1 = U_2$.

(7) Suppose
$$U = \{(x, x, y, y) \in \mathbb{F}^4 : x, y \in \mathbb{F}\}.$$
Find a subspace $W$ of $\mathbb{F}^4$ such that $\mathbb{F}^4 = U \oplus W$.

(8) Consider the vector space $V = \mathbb{R}^{\mathbb{R}}$ of functions functions $f : \mathbb{R} \to \mathbb{R}$. A function $f : \mathbb{R} \to \mathbb{R}$ is called *periodic* if there exists a positive number $p$ such that $f(x) = f(x + p)$ for all $x \in \mathbb{R}$. Is the set of periodic functions from $\mathbb{R}$ to $\mathbb{R}$ a subspace of $V$? Explain.

(9) Consider the vector space $V = \mathbb{R}^{\mathbb{R}}$ of functions $f : \mathbb{R} \to \mathbb{R}$. Let $U_o \subset V$ and $U_e \subset V$ be the sets of odd and even continuous functions respectively.
**Recall:** $f : \mathbb{R} \to \mathbb{R}$ is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$, and is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.
   (a) Show that for any $f \in V$,
   $$g(x) = \frac{f(x) + f(-x)}{2} \text{ is even, and } h(x) = \frac{f(x) - f(-x)}{2} \text{ is odd.}$$
   (b) Prove that $U_o$ and $U_e$ are subspaces of $V$.
   (c) Show that $V = U_o \oplus U_e$.