

MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 1
DR. ABDUL-RAHMAN

SUMMER 2019

- **Submit questions 1, 2(a), and 4 for grading.**
- **Due on: Monday 05-20-2019.**
- **You will have a quiz from the following questions on the due date.**

- (1) Write the following system of linear equations as an equation for a single function $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ for appropriate choices of $m, n \in \mathbb{Z}_+$,

$$\begin{aligned}x + 2y - 3z &= 4 \\x + 3y + z &= 11 \\2x + 5y - 4z &= 0 \\x + y + z &= 22\end{aligned}$$

- (2) Solve the following equations for z a complex number:
- (a) $z^3 - 4i = 0$.
 - (b) $z^6 + 8 = 0$.
- (3) Find $r > 0$ and $\theta \in [0, 2\pi)$ such that $(1 - i)/\sqrt{2} = re^{i\theta}$.
- (4) Show that for any $z \in \mathbb{C}$
- (a) $z\bar{z} = |z|^2$.
 - (b) $\operatorname{Re}z \leq |z|$ and $\operatorname{Im}z \leq |z|$.
- then use them to prove the triangle inequality

$$|z_1 + z_2| \leq |z_1| + |z_2|, \text{ for all } z_1, z_2 \in \mathbb{C}.$$

- (5) Let $z, w \in \mathbb{C}$. Prove the parallelogram law

$$|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2).$$

- (6) Read Section 2.3.4 then prove Theorem 2.3.5: parts (1) and (2).
- (7) Compute the real and the imaginary parts of e^{e^z} for $z \in \mathbb{C}$.
- (8) Given any complex number $\alpha \in \mathbb{C}$, show that the coefficient of the polynomial

$$(z - \alpha)(z - \bar{\alpha})$$

are real numbers.

- (9) Let $z, w \in \mathbb{C}$ with $\bar{z}w \neq 1$ such that either $|z| = 1$ or $|w| = 1$. Prove that

$$\left| \frac{z - w}{1 - \bar{z}w} \right| = 1$$

- (10) Given a polynomial $p(z) = a_n z^n + \dots + a_1 z + a_0$ with complex coefficients, define the **conjugate** of $p(z)$ to be the new polynomial

$$\bar{p}(z) = \bar{a}_n z^n + \dots + \bar{a}_1 z + \bar{a}_0.$$

- (a) Prove that $\overline{p(z)} = \bar{p}(\bar{z})$.
- (b) Prove that $p(z)$ has real coefficient if and only if $\bar{p}(z) = p(z)$.
- (c) Given polynomials $p(z)$, $q(z)$, and $r(z)$ such that $p(z) = q(z)r(z)$, prove that $\bar{p}(z) = \bar{q}(z)\bar{r}(z)$.