Test 2 **Introduction to Linear Algebra** **June 23, 2017**

MA 313

Name: ______________________ **Signature:** ______________________

SHOW ALL YOUR WORK!

1. [20 points] Given that

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

with $\det(A) = 5$, Find each of the following (explain)

(a) $\det 2A^{-1}A^T$.

(b) $\begin{vmatrix} a+2d & b+2e & c+2f \\ d & e & f \\ g & h & i \end{vmatrix}$.

(c) $\begin{vmatrix} a & b & c \\ 3a+4d & 3b+4e & 3c+4f \\ -g & -h & -i \end{vmatrix}$.

(d) $\begin{vmatrix} e & d & f \\ b & a & c \\ h & g & i \end{vmatrix}$.

2. [20 points] Let

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -3 & 5 \end{bmatrix}$$

(a) Find the determinant of A.

(b) Find all the eigenvalues of A and give their algebraic multiplicity.

3. [20 points] Let $M_{2\times 2}$ be the vector space of all 2×2 matrices and define $T : M_{2\times 2} \to M_{2\times 2}$ by $T(A) = A + A^T$, where

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

(a) Show that T is a linear transformation.

(b) Describe the kernel of T.

(c) Find a basis $\mathcal{B}$ of $M_{2\times 2}$, then find $[A]_{\mathcal{B}}$.

4. [20 points] Let $\mathbb{P}_2$ be the vector space of all polynomials of degree less than or equal 2. Let

$$p_1(t) = 1 + t^2, \quad p_2(t) = t - 3t^2, \quad p_3(t) = 1 + t - 3t^2.$$

(a) Use the coordinate vectors to show that $\mathcal{B} = \{p_1, p_2, p_3\}$ is a basis for $\mathbb{P}_2$.

(b) Find q in $\mathbb{P}_2$ given that $[q]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

5. [20 points] Find the Nul(A) and Col(A) where

$$A = \begin{bmatrix} 1 & 2 & 8 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$