Test 1 | **Introduction to Linear Algebra MA 313** | **May 26, 2017**

Name: ______________________ **Signature:** ______________________

SHOW ALL YOUR WORK!

1. [20 points] Row reduce each of the following matrices into the **row reduced echolon form**

$$A = \begin{bmatrix} 1 & 4 & 5 & -9 \\ -1 & -2 & -1 & 3 \\ -2 & -3 & 0 & 3 \\ 0 & -3 & -6 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 & -1 & 3 \\ 1 & 8 & -2 & 0 \end{bmatrix}$$

2. [10 points] Let $A = \begin{bmatrix} 0 & 1 & 4 \\ 2 & -3 & 2 \\ 4 & -8 & 12 \end{bmatrix}$, let $\mathbf{b} = \begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$, and let W be the span of the columns of A.

   (a) Is $\mathbf{b} \in W$? Explain.

   (b) Show that the third column of A is in W.

3. [15 points] Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined as

$$T(x_1, x_2) = (x_1 - 2x_2, x_1, x_1 + x_2)$$

   for every real numbers $x_1, x_2 \in \mathbb{R}$.

4. [15 points] Consider the system of linear equation that correspond to the following augmented matrix

$$\begin{bmatrix} 2 & 2 & 5 \\ -5 & h & k \end{bmatrix}$$

(a) Find all values of h so that the system has a **unique** solution.

(b) Find all values of h such that the columns of the coefficient matrix are **linearly dependent**.

(c) Find all values of h and k such that the system has **no solution**.

5. [10 points] Let A be a 3×5 matrix. Suppose that the fifth column in the row reduced echolon form of A is the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(a) Describe geometrically the solution set of the homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$.

(b) Describe geometrically the solution set of the matrix equation $A\mathbf{x} = \mathbf{a}_2 + \mathbf{a}_3$, where $\mathbf{a}_2$ and $\mathbf{a}_3$ are the second and third columns of A.

6. [15 points] Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in **vector form**, where A is row equivalent to the following

$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7. [15 points] Mark each statement as **True** or **False**, explain.

- (   ) The columns of any 4×3 matrix are linearly independent.
- (   ) Every linear transformation is a matrix transformation.
- (   ) If A is 2×5 matrix and T is the matrix transformation $T\mathbf{x} = A\mathbf{x}$, then the domain of T is $\mathbb{R}^5$ and its range is subset of $\mathbb{R}^2$.
- (   ) If A is 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot be onto.
- (   ) The map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x_1, x_2) = (0, 0)$ for all $x_1, x_2 \in \mathbb{R}$ is a linear transformation.