Test 1 | **Analysis for Engineers** | **Feb 10, 2020**

MATH 322 | **Dr. Abdul-Rahman**

Name: _____ **Signature:** _____

You must **show all work** to receive credit.
The test includes five questions.
The questions in the test have different levels of difficulty, make sure you manage your time wisely.

Question	Grade
1	
2	
3	
4	
5	
Total	

1. [30 points] Let

$$u(x, y) = x^3 - 3xy^2 - 2x,$$

(a) Verify that $u(x, y)$ is harmonic.

(b) Find the harmonic conjugate $v(x, y)$ of $u(x, y)$ in $f(z) = u + iv$.

(c) Find $f'(z)$.

2. [20 points] Solve the following equations, write the solution in the form of $x + iy$.

   (a) $\cos z = 6$.

   (b) $z^3 - 4i = 0$.

3. [30 points] Write the following in the form $x + iy$.

   (a) $\sin(2 + 3i)$.

   (b) $\mathrm{Ln}(-2 + i)$.

   (c) $\sqrt[2]{1 + i}$.

4. [30 points] Let $f(z) = u(x, y) + iv(x, y)$ be an entire function.

   (a) Show that αf is an entire function for every $\alpha \in \mathbb{C}$.

   (b) Show that $g(z) = v(x, y) - iu(x, y)$ is entire.

   (c) Suppose that $u_x(x_0, y_0) = 5$ and $u_y(x_0, y_0) = -1$, find $f'(x_0 + iy_0)$.

5. [20 points] Determine whether each of the following functions is continuous at $z = 0$. Show your work. (Here $z = x + iy$.)

(a) $f(z) = \begin{cases} \frac{xy}{x^2+y^2} & z \neq 0 \\ 0 & z = 0. \end{cases}$

(b) $g(z) = \begin{cases} \frac{x+2iy}{x+iy} & z \neq 0 \\ 1 & z = 0. \end{cases}$