MATH 322: MATHEMATICAL ANALYSIS FOR ENGINEERS

HOMEWORK FOR SECTIONS 7.4, 7.5, 7.8.

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SPRING 2020

[1] Mark each statement as True or False, explain.

(a) If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{w}$ and $\mathbf{v}$.

(b) If none of the vectors in $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a multiple of one of the other vectors, then S is linearly independent.

(c) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a set of linearly dependent vectors, then each vector in S is in the span of the other vectors.

(d) If a set contains fewer vectors then there are entries in the vectors, then the set is linearly independent.

(e) If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of A span $\mathbb{R}^m$.

(f) If the columns of an $n \times n$ matrix are linearly independent then they span $\mathbb{R}^n$.

(g) If $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is a basis for a subspace W of $\mathbb{R}^n$, then $\{\mathbf{u_1}, \mathbf{u_2}, \alpha\mathbf{u_3}\}$ is also a basis for W for every $\alpha \in \mathbb{R}$.

(h) If $A^3 = 0$ then $\det(A) = 0$.

(i) If A is an $n \times n$ matrix and $\alpha \in \mathbb{R}$ then $\det(\alpha A) = \alpha \det(A)$.

[2] Find the value(s) of h for which the following vectors are linearly independent.

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}.$$

[3] Do the columns of A span $\mathbb{R}^3$? Explain

$$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -1 \\ 5 & 8 & 7 \end{bmatrix}$$

[4] Find the rank and nullity of each of the following matrices

$$\begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix}, \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}.$$

[5] Suppose that A is a 3×5 matrix with $\operatorname{rank}(A) = 3$, what is the dim $\operatorname{null}(A)$?

[6] If the subspace of all solutions of $A\mathbf{x} = \mathbf{0}$ has a basis consisting of three vectors and if A is a 5×7 matrix, what is rank(A)?

[7] If possible, construct a 3×4 matrix A such that dim null$(A) = 2$.

[8] If A is a 6×8 matrix, what is the smallest possible dimension of null(A)?

[9] Let A be a 5×6 matrix, and let $\mathbf{b}$ be any vector in $\mathbb{R}^5$. What you can say about the consistency of the system $A\mathbf{x} = \mathbf{b}$ if rank$(A) = 5$? and if rank$(A) = 4$? Explain.

[10] Find the inverses of the following matrices

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 5 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 3 & -5 \end{bmatrix}$$

[11] Find the determinant of the matrices

$$\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{bmatrix}$$

[12] Let A be a 4×4 matrix, with $\det(A) = -3$ and $\det(B) = 4$. Find the following
(a) $\det(AB)$.
(b) $\det(2A)$.
(c) $\det(A^T BA)$.
(d) $\det(A^3 B^{-1})$.

[13] Given that

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

with $\det(A) = 3$, Find each of the following (explain)
(a) $\det\left(2A^{-1}(A^T)^2\right)$.

(b) $\begin{vmatrix} a+2d & b+2e & c+2f \\ d+g & e+h & f+i \\ g & h & i \end{vmatrix}$.

(c) $\begin{vmatrix} 2a & 2b & 2c \\ 3a+4d & 3b+4e & 3c+4f \\ -g & -h & -i \end{vmatrix}$.

(d) $\begin{vmatrix} e & d & f \\ b & a & c \\ -h & -g & -i \end{vmatrix}$.