Test 4 **MATH 129** **April 25, 2018**

Name: _____ **Signature:** _____

Show all your work!

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1. [20 points]

   (a) Find the first three non-zero terms of the Taylor series for $f(x) = e^{x^2}$ around $x = 0$.

   (b) Find the first three non-zero terms of the Taylor series for $\int_0^x e^{t^2}\, dt$ about $x = 0$.

2. [10 points] Find the Taylor series of $g(x) = \dfrac{x}{1+3x}$ about $x = 0$, find its radius of convergence.

3. [10 points] Expand $\sqrt[3]{P+t}$ in terms of $\frac{t}{P}$ (Give the first four non-zero terms).

4. [10 points] A function has the following Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{n x^{2n+1}}{(2n+1)!}$$

Find $f^{(101)}(0)$.

5. [20 points] Consider the differential equation

$$\frac{d\theta}{dt} = k(\theta - 3)(4 - \theta), \quad k > 0.$$

(a) Find all equilibrium solutions of the differential equation.

(b) Classify the solution(s) in part (a) as stable or unstable.

6. [20 points] Find the solution of the following differential equations

   (a) $\dfrac{dy}{dx} + \dfrac{y}{2} = 0, \;\; y(0) = 10.$

   (b) $\dfrac{dy}{dx} = \dfrac{y}{2+x}, \;\; y(0) = 3.$

7. [10 points] In the following cases, use k as the proportionality constant with $k > 0$.

   (a) Write the differential equation whose solution is the temperature of a bottle of orange juice taken out of a 40^o F refrigerator and left in 65^o F.

   (b) The rate at which a drug leaves the bloodstream and passes into the urine is proportional to the quantity of the drug in the blood at that time. Write a differential equation for the quantity, Q, of the drug in the blood after t hours.