Test 3 **MATH 129** **April 4, 2018**

Name: _____ **Signature:** _____

Show all your work!

1. [30 points] Determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+5}{n^2+3n+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$

2. [20 points] Determine whether the following series are convergent or divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ 2^n}{3^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n! \ (n+1)!}{(2n)!}$

3. [15 points] Find the radius and interval of convergence for the following power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-5)^n}{2^n \; n^2}$$

4. [10 points] Determine whether the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$$

5. [10 points] Give an example in each of the following cases:

(a) A divergent bounded sequence.

(b) A power series that is convergent only at $x = 2$.

(c) A series for which the integral test cannot be used.

(d) A divergent alternating series.

(e) A divergent series $\sum a_n$ such that $\sum (a_n)^2$ is convergent.

6. [20 points] A water tank is in the shape of a right circular cone with hight 18 ft and radius 12 ft at the top. It is filled with water to a depth of 15 ft. Set up the integral that finds the work done in pumping all the water over the top of the tank (the density of water is 62.4 lb/ft^3).