

Test 2

Linear Algebra MA 413

April 10, 2017

Name: _____

Signature: _____

SHOW ALL YOUR WORK!

In all of the following questions, V is a finite dimensional vector space over a field \mathbb{F} .

1. [20 points] Show that the linear map $T : \mathbb{F}^4 \rightarrow \mathbb{F}^2$ is surjective if

$$\text{null}(T) = \{(x_1, x_2, x_3, x_4) \in \mathbb{F}^4 \mid x_1 = 3x_2, x_3 = x_4\}$$

2. [15 points] Suppose that U and W are subspaces of V . Prove that if

$$\dim(V) < \dim(U) + \dim(W)$$

then $U \cap W \neq \{0\}$.

3. [20 points] Let $V := \mathbb{C}_n[z]$ is the vector space over \mathbb{C} consisting of all polynomials with degree less than or equal to n . Let $T \in \mathcal{L}(V)$ be the differentiation map

$$T(q) = q'.$$

- Give a basis for V then find $M(T)$, i.e., the matrix representation of T , under that basis.
- Find all the eigenvalues of T .

4. [20 points] Let V and W be vector spaces over \mathbb{F} , and suppose that $T \in \mathcal{L}(V, W)$ is injective. Given a linearly independent list of vectors (v_1, \dots, v_n) of V , prove that the list $(T(v_1), \dots, T(v_n))$ is linearly independent in W .

5. [20 points] Let $T \in \mathcal{L}(V)$ be a linear map, and suppose that λ is an eigenvalue of T . Let $U = \text{range}(T - \lambda\mathbb{1})$.
- (a) Show that $\dim U < \dim V$.
 - (b) Prove that U is T -invariant.

6. [15 points] Suppose $T \in \mathcal{L}(V)$ and the set $B = \{v_1, v_2, \dots, v_n\}$ is a basis of V for which $M(T)$, the matrix corresponding to T in the basis B , is upper triangular. Prove that if T is invertible, then the matrix $M(T^{-1})$, the matrix corresponding to T^{-1} in the basis B , is also upper triangular.