HW #2

Thm 3.2.2 Part 3 p31 given $n \in \mathbb{Z}_+$, and any choice of $a_0, \dots, a_n \in \mathbb{C}$ with $a_n \neq 0$, define $f: \mathbb{C} \to \mathbb{C}$

$$f(z) = a_n z^n + \dots + a_1 z + a_0, \quad \forall z \in \mathbb{C}$$

then

there ~~ex~~ exist exactly $n+1$ complex numbers $w_0, \dots, w_n \in \mathbb{C}$ such that

$$f(z) = w_0(z - w_1) \dots (z - w_n), \quad \forall z \in \mathbb{C}.$$

Proof:

by induction:

- for $n = 1$

$$f(z) = a_0 + a_1 z, \quad a_1 \neq 0$$

$$= a_1\left(\frac{a_0}{a_1} + z\right) \quad \checkmark$$

$$w_0 = a_1, \quad w_1 = \frac{-a_0}{a_1}$$

- suppose the theorem is true for the case $n-1$, want to show that it is true for n

i.e. want to show that every polynomials with coeffints over $\mathbb{C}$ of degree "n" can be factored into n linear factors.

let f be a poly. of degree n, then by part 1 from theorem 3.2.2, $\exists$ a polynomial g of degree $n-1$ such that

$$f(z) = (z - w)\, g(z) \quad \text{——} \; 1$$

then using the inductive assumption, $\exists$ n complex numbers $w_0, w_1, \dots, w_n$ such that

$$g(z) = w_0(z - w_1) \dots (z - w_n) \quad \text{——} \; 2$$

and by substituting 2 back in 1 we get

$$f(z) = w_0(z - w)(z - w_1) \dots (z - w_n)$$

$\boxtimes$

$\boxed{\text{Calculational ex p34}}$

$\boxed{Q2}$ $\alpha \in \mathbb{C}$

$$(z-\alpha)(z-\bar{\alpha}) = z^2 - \bar{\alpha}z - \alpha z + \alpha\bar{\alpha}$$
$$= z^2 - (\alpha+\bar{\alpha})z + |\alpha|^2$$
$$= z^2 - 2Re(\alpha)z + |\alpha|^2$$

it is a poly with real coefficients.

Proof-Writing Ex: p34

$\boxed{Q2}$ $\boxed{a}$ prove that $\overline{p(z)} = \bar{p}(\bar{z})$

$$\overline{p(z)} = \overline{a_n z^n + \dots + a_1 z + a_0} = \overline{a_n z^n} + \dots + \overline{a_1 z} + \bar{a}_0$$
$$= \bar{a}_n \bar{z}^n + \dots + \bar{a}_1 \bar{z} + \bar{a}_0$$
$$= \bar{p}(\bar{z})$$

(b) $\boxed{\Rightarrow}$ suppose that $a_0, a_1, \dots, a_n \in \mathbb{R}$ then $\bar{a}_j = a_j \ \forall j$

$\Rightarrow \bar{p}(z) = p(z)$

$\boxed{\Leftarrow}$ suppose that $\bar{p}(z) = p(z)$

$\Rightarrow \bar{a}_n z^n + \dots + \bar{a}_1 z + a_0 = a_n z^n + \dots + a_1 z + a_0$

$\Rightarrow \bar{a}_j = a_j$ for all $j = 0, 1, \dots, n$

$\Rightarrow a_j$ are real

$\boxed{c}$ given that $p(z) = q(z)r(z) \Rightarrow p(\bar{z}) = q(\bar{z})r(\bar{z})$

$\Rightarrow \overline{p(\bar{z})} = \overline{q(\bar{z})}\ \overline{r(\bar{z})}$

$\Rightarrow$ (by part $\boxed{a}$) $\bar{p}(\bar{\bar{z}}) = \bar{q}(\bar{\bar{z}})\ \bar{r}(\bar{\bar{z}})$

$\Rightarrow \bar{p}(z) = \bar{q}(z)\ \bar{r}(z)$