Test 1 **Linear Algebra** **May 31, 2019**

MATH 413/513 **Dr. Abdul-Rahman**

Name: _____ **Signature:** _____

Show all your work!

(1) [20 points] Write the following system of linear equations as an equation for a single function $f : \mathbb{R}^n \to \mathbb{R}^m$ for appropriate choices of $m, n \in \mathbb{Z}_+$,

$$\begin{aligned} x + 2y - 3z + w &= 11 \\ x + 3y + z - w &= -1 \\ y + w &= 0 \end{aligned}$$

(2) [20 points] Solve the following equation for $z \in \mathbb{C}$,

$$z^3 - i = 1.$$

(3) [20 points] Let U_1 and U_2 be two subspaces of a vector space $V(+, \cdot)$. In each of the following, prove or give a counter example:

(a) $U_1 \cap U_1$ is a subspace of V.

(b) $U_1 \cup U_2$ is a subspace of V.

(c) $U_1 - U_2$ is a subspace of V, where

$$U_1 - U_2 := \{u_1 + (-u_2);\ u_1 \in U_1,\ u_2 \in U_2\}.$$

(4) [20 points] Let

$$U := \{(x, x, 0) \in \mathbb{R}^3;\ x \in \mathbb{R}\}$$

(a) Show that U is a subspace of $\mathbb{R}^3$.

(b) Find a subspace W of $\mathbb{R}^3$ such that $\mathbb{R}^3 = U \oplus W$.

(5) [20 points] Consider the vector space $\mathbb{R}^{[0,1]}$ of functions $f : [0, 1] \to \mathbb{R}$. Define

$$S = \left\{ f \in \mathbb{R}^{[0,1]};\ f \text{ is continuous on } [0, 1] \text{ and } \int_0^1 f(x)\, dx = 0 \right\}$$

Show that S is a subspace of $\mathbb{R}^{[0,1]}$.

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