

MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 2

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SUMMER 2019

- **Submit question 8 for grading.**
- **Due on: 05/29/2020.**
- **You have a quiz on 05/29/2020.**

(1) The empty set fails to satisfy only one of the requirements of vector spaces. Which one? Explain.

(2) Let $\mathcal{P}(\mathbb{F})$ be the vector space of all polynomials with coefficients from the Field \mathbb{F} . Let

$$W := \mathcal{P}(\mathbb{F}) \times \mathbb{F}^2 = \{(q, u); q \in \mathcal{P}(\mathbb{F}), u \in \mathbb{F}^2\}.$$

Equip W with a vector addition $(+)$ and scalar multiplication (\cdot) in such a way that W becomes a vector space, and prove that $W(+, \cdot)$ is a vector space over \mathbb{F} .

(3) Suppose U_1 and U_2 are subspaces of the vector space V . Prove that the intersection $U_1 \cap U_2$ is a subspace of V .

(4) Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.

(5) Prove or give a counterexample: If U_1, U_2 , and W are subspaces of V such that

$$U_1 + W = U_2 + W.$$

Then $U_1 = U_2$.

(6) Prove or give a counterexample: If U_1, U_2 , and W are subspaces of V such that

$$U_1 \oplus W = U_2 \oplus W.$$

Then $U_1 = U_2$.

(7) Suppose

$$U = \{(x, x, y, y) \in \mathbb{F}^4; x, y \in \mathbb{F}\}.$$

Find a subspace W of \mathbb{F}^4 such that $\mathbb{F}^4 = U \oplus W$.

(8) Consider the vector space $V = \mathbb{R}^{\mathbb{R}}$ of functions $f : \mathbb{R} \rightarrow \mathbb{R}$. Let $U_o \subset V$ and U_e be the sets of odd and even functions; respectively.

Recall: $f : \mathbb{R} \rightarrow \mathbb{R}$ is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$, and is *even* if $f(-x) = f(x)$ for all $x \in \mathbb{R}$.

(a) Show that for any $f \in V$,

$$g(x) = \frac{f(x) + f(-x)}{2} \text{ is even, and } h(x) = \frac{f(x) - f(-x)}{2} \text{ is odd.}$$

(b) Prove that U_o and U_e are subspaces of V .

(c) Show that $V = U_o \oplus U_e$.