MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 1

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SUMMER 2020

- **Submit questions 1 and 4 for grading (Upload to D2L).**
- <u>**Due on:**</u> **Monday 05/25/2019 before class.**
- **You will have a quiz from the following questions on Monday 05/25/2020.**

(1) Write the following system of linear equations as an equation for a single function $f : \mathbb{R}^n \to \mathbb{R}^m$ for appropriate choices of $m, n \in \mathbb{Z}_+$,

$$\begin{aligned} x + 2y - 3z &= 4 \\ x + 3y + z &= 11 \\ 2x + 5y - 4z &= 0 \\ x + y + z &= 22 \end{aligned}$$

(2) Solve the following equations for z a complex number:
  (a) $z^3 - 4i = 0$.
  (b) $z^6 + 8 = 0$.

(3) Find $r > 0$ and $\theta \in [0, 2\pi)$ such that $(1-i)/\sqrt{2} = re^{i\theta}$.

(4) Show that for any $z \in \mathbb{C}$
  (a) $z\overline{z} = |z|^2$.
  (b) $\text{Re}z \leq |z|$ and $\text{Im}z \leq |z|$.

then use them to prove the triangle inequality

$$|z_1 + z_2| \leq |z_1| + |z_2|, \text{ for all } z_1, z_2 \in \mathbb{C}.$$

(5) Let $z, w \in \mathbb{C}$. Prove the parallelogram law

$$|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2).$$

(6) Read Section 2.3.4 then prove Theorem 2.3.5: parts (1) and (2).

(7) Compute the real and the imaginary parts of e^{e^z} for $z \in \mathbb{C}$.

(8) Given any complex number $\alpha \in \mathbb{C}$, show that the coefficient of the polynomial

$$(z - \alpha)(z - \overline{\alpha})$$

are real numbers.

(9) Let $z, w \in \mathbb{C}$ with $\overline{z}w \neq 1$ such that either $|z| = 1$ or $|w| = 1$. Prove that

$$\left| \frac{z - w}{1 - \overline{z}w} \right| = 1$$

(10) Given a polynomial $p(z) = a_n z^n + \ldots + a_1 z + a_0$ with complex coefficients, define the **conjugate** of $p(z)$ to be the new polynomial

$$\overline{p}(z) = \overline{a_n} z^n + \ldots \overline{a_1} z + \overline{a_0}.$$

  (a) Prove that $\overline{p(z)} = \overline{p}(\overline{z})$.
  (b) Prove that $p(z)$ has real coefficient if and only if $\overline{p}(z) = p(z)$.
  (c) Given polynomials $p(z)$, $q(z)$, and $r(z)$ such that $p(z) = q(z)r(z)$, prove that $\overline{p}(z) = \overline{q}(z)\overline{r}(z)$.