MATH 413/513 (LINEAR ALGEBRA)

HOMEWORK 7 - SUMMER 2018

DR. ABDUL-RAHMAN

Due on: **Friday 06-22-2018.**

(1) Let $\mathbb{F}_m[z]$ denote the vector space of all polynomials with degree less than or equal to $n \in \mathbb{Z}_+$ and having coefficient over $\mathbb{F}$, and suppose that $p_0, p_1, \ldots, p_m \in \mathbb{F}_m[z]$ satisfy $p_j(1) = 0$. Prove that $(p_0, p_1, \ldots, p_m)$ is a linearly dependent list of vectors in $\mathbb{F}_m[z]$.

(2) Let V be a finite dimensional vector space over the field $\mathbb{F}$, and suppose that $P \in \mathcal{L}(V)$ has the property that $P^2 = P$. Prove that $V = \text{null}(P) \oplus \text{range}(P)$

(3) Solve for x the following equation

$$\det\left(\begin{bmatrix} x & -1 \\ 3 & 1-x \end{bmatrix}\right) = \det\left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & x & -6 \\ 1 & 3 & x-5 \end{bmatrix}\right)$$

(4) Let A be a square matrix. Prove that A is invertible if and only if $A^T A$ is invertible.

(5) Suppose that V is a real inner product space
  (a) Show that if $u, v \in V$ have the same norm, then $u + v$ is orthogonal to $u - v$.
  (b) Use part (a) to show that the diagonals of a rhombus are perpendicular to each other.

(6) Suppose that $T \in \mathcal{L}(V)$ is such that $\|Tv\| \leq \|v\|$ for all $v \in V$. Prove that $(T - 2\mathbb{1})$ is invertible.

(7) Suppose $(V, \langle \cdot, \cdot \rangle)$ is an inner product space. Let $T \in \mathcal{L}(V)$ is an injective operator on V. Define $\langle \cdot, \cdot \rangle_1$ by

$$\langle u, v \rangle_1 := \langle Tu, Tv \rangle, \text{ for all } u, v \in V.$$

Show that $\langle \cdot, \cdot \rangle_1$ is an inner product on V.