MATH 413/513 (LINEAR ALGEBRA)

HOMEWORK 6 - SUMMER 2018

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<u>Due on:</u> Thursday 06-14-2018.

In the following, V is a finite dimensional vector space over the field $\mathbb{F}$.

(1) Let $S, T \in \mathcal{L}(V)$ and denote by $\mathbb{1}$ map on V. Prove that $T \circ S = \mathbb{1}$ if and only if $S \circ T = \mathbb{1}$.

(2) Let $n \in \mathbb{Z}_+$ be a positive integer and $T \in \mathcal{L}(\mathbb{F}^n)$ be defined by

$$T(x_1, \ldots, x_n) = (x_1 + \ldots + x_n, \ldots, x_1 + \ldots + x_n)$$

for every $x_1, x_2, \ldots, x_n \in \mathbb{F}$. Compute the eigenvalues and associated eigenvectors for T.

(3) Let $T \in \mathcal{L}(V)$ and let U_1 and U_2 be two T-invariant subspaces of V.
   (a) Prove that $U_1 + U_2$ is T-invariant.
   (b) Prove that $U_1 \cap U_2$ is T-invariant.

(4) Suppose that $T \in \mathcal{L}(V)$ has the property that every $v \in V$ is an eigenvector for T. Prove that T must then be a scaler multiple of the identity function on V.

(5) Let V be a finite dimensional over $\mathbb{C}$, $T \in \mathcal{L}(V)$, and $p(z) \in \mathbb{C}[z]$ be a polynomial. Prove that $\lambda \in \mathbb{C}$ is an eigenvalue of the linear operator $p(T) \in \mathcal{L}(V)$ if and only if T has an eigenvalue $\mu \in \mathbb{C}$ such that $p(\mu) = \lambda$.